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ROBUSTNESS OF ADAPTATION IN CONTROLLED SELF-ADJUSTING CHAOTIC SYSTEMS

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It was recently shown that self-adjusting systems adapt to the edge of chaos. We study the robustness of that adaptation with respect to a controlling force. We first use numerical simulations in a modified logistic map. With these, we find that, if the controlling force has a target value of the parameter that leads to periodic dynamics, the control is successful, even for very small controlling forces. We also find, however, that if the target value for the parameter leads to chaotic dynamics, the parameter resists the control and adaptation to the edge of chaos is still observed. When the controlling force is very strong, adaptation to the edge of chaos is weaker, but still present in the system. We also perform experiments with a self-adjusting Chua circuit and find the same behavior. We quantify these results with a measurement of the robustness of the adaptation as a function of the strength of the controlling force. The control used can be expressed either as a parametric control or as an additive, closed-loop control.

Keywords: Adaptive behavior; robustness; control of chaos; self-organization.

1. Introduction

In the field of complex systems, there has been an interest in showing that adaptive systems will adapt to a new state at the boundary of chaos and order, called the edge of chaos [1]. N.H. Packard [2] first showed that this effect occurred for a population of cellular automata rules evolving with a genetic algorithm, though the conclusions drawn from this work has come under dispute [3]. Pines and Hübler [4] studied two competitive, adaptive agents which used both control and modeling to predict the behavior of the logistic map and found that, over time, the agents use a control which places the logistic map at the edge of chaos. The edge of chaos occupies a

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prominent position because it has been found to be not only an optimal setting for control of a system [5], but also an optimal setting under which a physical system can support primitive functions for computation [6], though once again, this claim has been disputed [7]. From a dynamical systems perspective, the edge of chaos has been shown to exhibit critical behavior [8], including $1/f$-type noise [9]. This has led to comparisons between the idea of adaptation to the edge of chaos and self-organized criticality [10].

Recently, we have proposed a new model for adaptive systems which treats the control parameters of the system as slowly varying, rather than constant. Adaptive systems of this type, which we refer to as self-adjusting systems [5], have recently been shown to exhibit adaptation to the edge of chaos [11]. The parameters in a self-adjusting system have a simple, over-damped motion without attractor. They are distinguished from the dynamical variables through a separation of time scales [12], that is, they change much more slowly than the dynamical variables. The parameters dynamics are governed by a low-pass filtered feedback with DC cutoff from the dynamical variables of the system itself. Previous numerical results showed that, for the logistic map, the parameter changed in an apparently random fashion until it reached the edge of chaos [11]. Adaptation to the edge of chaos is observed when, in the limiting state, the parameter has a high probability of being at a value in which, with a small perturbation, the resulting dynamics changes from chaotic to non-chaotic, or vice-versa. Other studies have investigated logistic maps where the parameters vary as the result of a coupling to another logistic map [13] or to a hidden system [14] and found qualitatively different behavior than observed in self-adjusting systems.

In Sec. 2 of this letter, we introduce a modified self-adjusting logistic map subject to a control force. With numerical studies, we find that when the target value leads to chaotic dynamics, the parameter still leaves the chaotic regime and that adaptation to the edge of chaos is still observed. However, if the target value leads to periodic dynamics, the parameter follows the target, even for small controlling forces. We quantify the robustness by measuring the precision of adaptation as a function of the applied controlling force. In Sec. 3, we apply a similar forcing function to an experimental setup, the self-adjusting Chua circuit [15]. We find the behavior is similar to that in the modified logistic map. Finally, we summarize our results and discuss future work.

2. Robustness in a Controlled Self-Adjusting Logistic Map

To study the effect of a parametric forcing on a self-adjusting system, we look at a modified self-adjusting logistic map:

$$\begin{aligned} x_{n+1} &= 3.8(1-b_n^2)x_n(1-x_n), \qquad 0 \le x_n \le 1, \\ b_{n+1} &= b_n + L(x) + F(n, b_n), \qquad -1 \le b_n \le 1, \end{aligned} \tag{1}$$

where x is the variable, b is the parameter, $L(x)$ is a low-pass filtered feedback of the x dynamics, and F is an external forcing function. This system is the logistic map, with a simple change to the parameter. The parameter ranges from -1 to 1. The parameter dependence of the map is determined by the traditional logistic map parameter, a, with $a = 3.8(1 - b_n^2)$, with the logistic map given by $x_{n+1} =$

$ax_n(1 - x_n)$. When this is greater than 3.569, the dynamics are mostly chaotic. This translates to values of b between -.247 and .247. Therefore, this system has a chaotic band in the middle of the range of the parameter, similar to our experimental system in Sec. 3.

To study the dynamics of the system, a low-pass filter must be chosen for the feedback term in Eq. (1). The filter used in the previous studies of the logistic map [11] is used. This filter can be calculated in the numerical simulations with a Fourier analysis of the time series for x_n. If N time steps are used, the Fourier sine and cosine coefficients are given by: $c_{nk} = (2/N)\sum_{t=0}^{N-1} x(t+n-N+1)\sin(2\pi kt/N)$, $d_{n0} = (1/N)\sum_{t=0}^{N-1} x(t+n-N+1)$,and $d_{nk} = (2/N)\sum_{t=0}^{N-1} x(t+n-N+1)\cos(2\pi kt/N)$ for $k = 1, 2, ..., (N-1)/2$ where k is the frequency. If N is odd, an extra term is needed: $d_{n(N+1)/2} = (1/N)\sum_{t=0}^{N-1} x(t+n-N+1)\cos(\pi(N+1)t/N)$. To make a low-pass filter with a very low cutoff frequency and DC cutoff, only the terms c_{n1} and d_{n1} would be kept. The back transformation would then become:

$$\bar{x}_n = c_{n1}\sin(\frac{2\pi n}{N}) + d_{n1}\cos(\frac{2\pi n}{N}). \tag{2}$$

If the forcing is only applied once every N steps, and is evaluated when n is a multiple of N, L_n becomes simply:

$$L_n = \begin{cases} \epsilon\bar{x}_N = \epsilon d_{n1} & \text{if } n = iN, \\ 0 & \text{if } n \neq iN, \end{cases} \qquad i = 1, 2, 3, \ldots. \tag{3}$$

With the low-pass filter fully specified, we now want to study the effect of the controlling force, F. In particular, we'll investigate the case when F is trying to control the parameter to a target value, B_n:

$$F(n, b_n) = -k(b_n - B_n) \tag{4}$$

where k is the strength of the forcing. It should be noted that, with the appropriate rearrangement of terms, this control method is exactly equivalent to a closed-loop control applied to the variable, with a target dynamics specified by a logistic map with parameter B_n. To fully specify the dynamics, the last issue to look at is the target value, B_n which may depend on time. In this letter, we specifically look at target values of the form:

$$B_{n+1} = B_n + \delta \tag{5}$$

where δ is a small drift term. In this way, we can start with the parameter so that the system is in the periodic regime and then force it to move through the chaotic band and back into periodicity. The results are shown in Fig. 1. In this case, the forcing is very small, $k = 0.0005$. There are two points to notice. First, when the target value of the parameter would lead to periodic motion in the variable, the control is successful. Second, when the target value of the parameter leads to chaotic motion in x, the control fails and the system remains in the periodic regime. This can be seen in both the time series plot for the parameter, as well as in the histogram. There exists large peaks in the probability distribution of the parameter at the edge of chaos, $b = \pm.247$. The control has near zero effect on the system in the chaotic regime, and the probability of finding the parameter there is very close to zero.

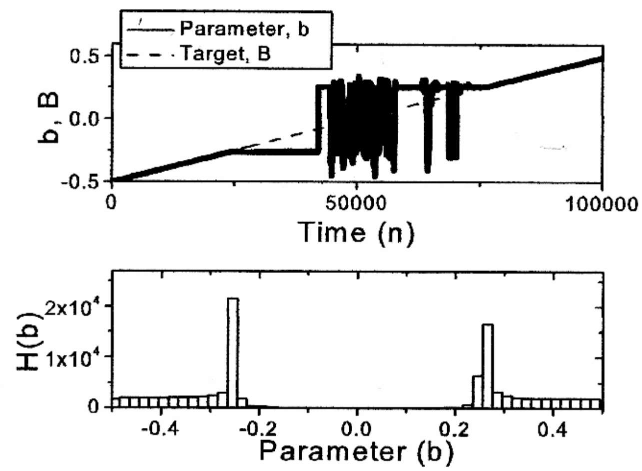

Fig. 1. Adaptation with weak forcing. Here, we use a very small value for the size of the forcing function, $k = 0.005$. The parameter spends most of its time at the edge of chaos and only enters the chaotic regime for small periods of time. The top plot shows the parameter and target as a function of time. The bottom plot shows a histogram of the parameter values, $H(b)$. If the control was perfect, the histogram would be flat across all parameter values.

Robustness can be measured as the precision of adaptation as a function of the total parameter variation in the system [16]. In this study, we only vary the strength of the forcing function. Other parameters in the system come in the form of the initial parameter value, which was studied in [11] or the specific form of the low-pass filter, discussed in [17]. For both of those cases, adaptation to the edge of chaos still occurred. In Fig. 2, we show the result of varying the strength of the control force. Even in the case of a very strong control force, we find that the system will adapt to the edge of chaos a significant percentage of the time. The combination of all of these results lead us to the conclusion that adaptation to the edge of chaos is a robust feature of self-adjusting systems. We wish, however, to study this experimentally, and in the next section present the results of experimental work on the self-adjusting Chua circuit.

3. Robustness in the Self-Adjusting Chua Circuit

Experimental verification of adaptation to the edge of chaos in self-adjusting systems has been shown for a self-adjusting Chua Oscillator [15]. We have used the same experimental setup to look at the robustness of adaptation in the self-adjusting Chua circuit. The circuit diagram is shown in Fig. 3. The two key elements of the circuit are the nonlinear resistor, described elsewhere [15, 18] and the variable resistor. The value of the variable resistor acts as the control parameter in the system, determining what dynamics is observed in the system. In the self-adjusting Chua Oscillator, the variable resistor is changed through a low-pass filtered feedback from the dynamical variable, in this case the voltage V_{C1} across one of the two capacitors in the circuit, $C1$:

$$R_{t+1} = R_t + L(V_{C1}) \tag{6}$$

where R is the variable resistor and L is a low pass filter on the V_{C1} dynamics. The implementation of the filter is described in [15], but is again determined through a Fourier analysis.

The dynamics of the Chua circuit, including the resistance dependence of the oscillations are described in [15]. Here, we wish to study the effect of a parametric control force in this experimental system. Once again, if we apply a forcing function to the parameter, to control it to a target dynamics, T, we have the following equation which describes the dynamics of the self-adjusting parameter:

$$R_{t+1} = R_t + L(V_{C1}) + k(T_{t+1} - R_t) \tag{7}$$

where R is the variable resistor, L is a low pass filter on the V_{C1} dynamics, T is the target value of the resistance, and k is a scale constant reflecting the strength of the forcing function for the parameter. The target dynamics are given by:

$$T_{t+1} = T_t - \delta \tag{8}$$

where δ is a small drift term.

Shown in Fig. 3, we show the results from experiment. The edge of chaos in the Chua's circuit is at $R = 1760\Omega$, where there is a period-doubling transition to chaos. At $R = 1400\Omega$, the dynamics makes a transition to a large limit cycle dynamics, another edge of chaos. However, there is a hysteresis effect in the Chua circuit. If R is lowered below 1400Ω, then increasing R does not cause the system to become chaotic. Only until a very large R is reached and then R is once again lowered into the chaotic regime does chaos reappear in the system. In the figure, what happens is that the parameter avoids the chaotic regime, until it reaches the large limit cycle at time $n \approx 160000$. Then, because the dynamics becomes regular, it no longer resists the forcing function and the system returns to the target dynamics. However, chaos in the system is avoided, and the highest probability of finding the system still remains at the edge of chaos, near $R = 1760\Omega$.

4. Conclusions

As discussed in our previous paper [11], adaptation to the edge of chaos in our model is a direct result of the combination of the low-pass filtered feedback and a fundamental difference between chaotic and periodic motion. Periodic motion has a finite recurrence time and, hence, a lowest frequency component. Chaotic motion, however, has an infinite recurrence time which means that there does not exist a lowest frequency component to its power spectrum. In the context of our system, if the motion of the variable is periodic, no signal will pass through the low-pass filter. If the motion is chaotic, a noisy signal will always be present. Therefore, the self-adjusting parameter will have a random walk like motion when the variable's dynamics are chaotic and no motion when the variable's dynamics are periodic.

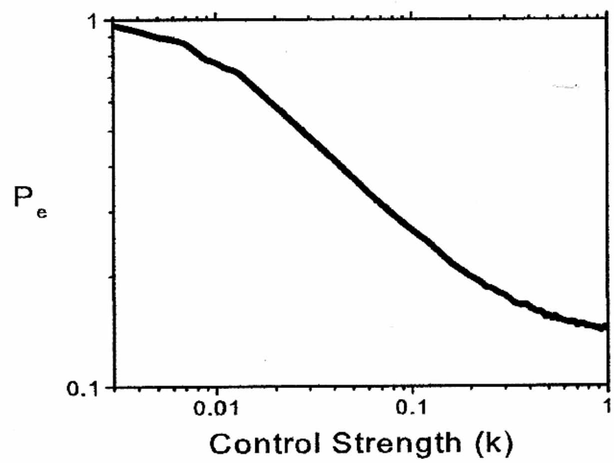

Fig. 2. Robustness of adaptation to the edge of chaos. The percentage of initially chaotic parameter values which, asymptotically, are found at the edge of chaos is measured as a function of the strength of the control force. The system successfully adapts to the edge of chaos a significant fraction of the time, even for very large controlling forces.

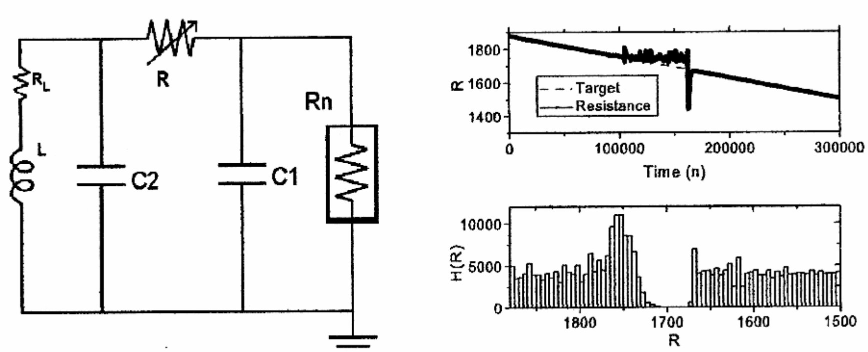

Fig. 3. Adaptation with small forcing in the Chua's circuit. The circuit diagram is shown on the left. Rn is a nonlinear resistor. In our experimental circuit, $L = 91.9$ mH, $R_L = 17.3\Omega$, $C_1 = 52.31$ nF, $C_2 = 567$ nF, and R varies from 1350 Ω to 1950 Ω. For the plots on the right, $k = 0.001$ and $\delta = 0.005\Omega$. There exists a peak at the edge of chaos in the histogram, $H(R)$, $R = 1760\Omega$. After R reaches a value of 1400Ω the dynamics are a large limit cycle, with a hysteresis effect when R is then increased, causing the system to stay periodic.

The choice of a low-pass filter is governed by the separation of time scales between the variable's dynamics and the parameter's dynamics. As such large separation of time scales is a common occurence in natural systems, we expect that adaptation to the edge of chaos should be present in many of those systems. This picture is not complete in the present context, however. With the addition of the controlling force we presented in this paper, the parameter's motion is no longer a simple random walk. However, as long as the controlling force remains small, the accuracy of the adaptation is high. This robustness is important if the mechanism for adaptation is to still work in more complicated systems.

Finally, it should be noted that the low-pass filtered feedback in itself can be thought of as a method for controlling chaos. After the initial work in controlling chaos [19, 20], controlling systems to move them from chaotic dynamics to periodic dynamics became a very important topic. Low-pass filtered feedback from the dynamical variables to the parameters represents a novel way to do this. Its chief advantage is that no knowledge of the underlying dynamics is needed. The drawbacks are that large changes to the control parameter may be necessary and the time for the system to leave the chaotic regime can be long. However, the ease of its application and the robust result may prove useful in some systems.

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