Description of transient states of von Kármán vortex streets by low-dimensional differential equations

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Aperiodic time series of hot-wire signals can be described as trajectories in a state space representation. The flow vector field is calculated by numerical differentiation of these trajectories and then each component of the flow vector field is approximated by a polynomial of order p. This approximation provides a model for the dynamics of the von Kármán vortex street by a low-dimensional system of ordinary differential equations. At a Reynolds number of 114 a compact description of the complex dynamics of the vortex street by a set of only ten parameters can be obtained. It will be shown that these parameters are independent of the probe position for distances greater than two-and-one-half cylinder diameters.

During the last few years a large number of attempts have been made to model a von Kármán vortex street. Benaroja and Lepore[1] assumed the van der Pol oscillator as a model for the vortex street. To describe the behavior of a sound-stimulated vortex street the Landau equation was used by Provansal, Mathis, and Boyer.[2] Olinger and Sreenivasan[3] experimentally found chaotic behavior. From all of this and from the knowledge of the region of frequency entrainment of the vortex street for sinoidal perturbations (Detemple[4]), it can be concluded that the dynamics of the vortex street can be modeled by a system of low-dimensional differential equations. Roesch, Eckelmann, and Hübler[5] could verify quantitatively a low-dimensional model for the regular range of the von Kármán vortex street (Roshko[6]) by using the method of construction of differential equations from experimental data. This method was developed by Cremers and Hübler[7] and further developed by Crutchfield and McNamara[8] and Farmer and Sidorowich.[9] The method of Cremers and Hübler has many advantages. First, it describes the aperiodic and transient dynamics of a system only by a few parameters, which is not possible by using Fourier coefficients. Second, with this knowledge of the constructed differential equation it is possible to control a complex system with special nonlinear driving forces[10] or to stimulate a nonlinear system resonantly.[11] The general problem with this method is that the coefficients of the constructed differential equation are correlated, i.e., there exists no clear solution of the adaptation of the polynomial to the flow vector field. Only if the region of interest of the phase space is filled up homogeneously with experimentally obtained data, and if the approximated polynomials are orthogonal, can a definite solution of the fit be achieved. To overcome these problems a knowledge of the transient behavior of the vortex street is of extreme importance. To obtain this information the vortex street was stimulated by sound to a slightly different velocity fluctuation amplitude. The stimulation frequency was chosen equal to or a few percent different from the Strouhal frequency in the region of frequency entrainment. After cutting off the sound the stimulated system returns to the natural state and the transients that are necessary for the investigations can be achieved. In earlier investigations,[5] the region immediately after the sound cutoff was considered as the transient state. However, the results presented here clearly show that an intermediate region with a higher frequency is observed between the cutoff and the real transience.

The measurements were carried out in the open-circuit-type wind tunnel described by Detemple and Eckelmann[12] at a velocity $U_\infty \approx 1.25$ m/sec and a free-stream turbulence level of about 0.2%. The nozzle diameter of this tunnel is 180 mm. A circular cylinder, which was mounted horizontally directly at the nozzle exit, had a diameter $d = 1.5$ mm that yields a Reynolds number $\mathrm{Re} = U_\infty d/\nu = 114$. A hot-wire probe was placed in the wake of the cylinder at various $\tilde{x} = x/d$ locations downstream, $y/d = 1$ to one side and halfway between the suspension points of the cylinder. The velocity signal was amplified, digitized by a 12 bit A/D converter and sampled at 10 kHz. Two loudspeakers working 180° out of phase were placed directly above and below the circular cylinder outside the flow to superimpose sound. Note that this loudspeaker arrangement is different from the

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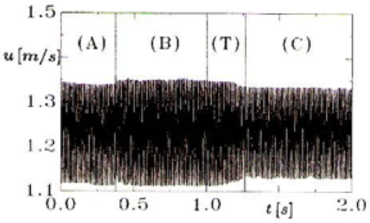

FIG. 1. Typical hot-wire signal from a von Kármán vortex street at Re = 114. (A): Vortex street stimulated by sound; (B): intermediate region after cutting off the sound; (T): transient state of the vortex street; (C): natural vortex street.

one used by Detemple and Eckelmann.

A typical experimental time series showing the transient behavior from the stimulated to the natural state of the vortex street is depicted in Fig. 1. Four different parts, which can be attributed to different physical processes, are marked. The power spectra for three of these parts are shown in Fig. 2. Part (A) in Fig. 1 represents the vortex street stimulated by sound with parallel vortex shedding[12] at the sound frequency ($f_s = 119.6$ Hz). The sound frequency was chosen slightly higher than the natural Strouhal frequency ($f_{\mathrm{St}} = 119.0$ Hz) in order to increase the velocity fluctuation amplitude. This method of increasing the velocity fluctuation amplitude by stimulating the vortex street with a slightly different frequency in order to measure the transient behavior was developed by the authors. It means the increase in velocity fluctuation amplitude by this method can only be successfully achieved when the natural case, without external excitation, is slantwise vortex shedding. Part (B) in Fig. 1 defines the intermediate region between cutting off the sound and the beginning of the transient state. Note that in this region, right after cutting off the sound, the shedding frequency jumps from f_s to $f_{\max} = 124.5$ Hz (Fig. 2 dashed line). Without the sound excitation, the unforced parallel shedding of part (B) has a slightly higher frequency than the slantwise shedding of part (C), which is consistent with recent experiments.[13,14] Some transient experiments with a towing tank by Williamson[15] show that the parallel shedding remains until slantwise shedding, originating from the ends of the cylinder, has propagated to the central region of the cylinder. When his cylinder was set into motion at the beginning parallel shedding with the frequency f_p was observed. The following oblique shedding with an angle φ with respect to the cylinder starts at one of the suspension points of the cylinder and that leads to the lower shedding frequency f_{St}. Between the two frequencies Williamson found the relation:

$$f_p = f_{\mathrm{St}} / \cos \varphi. \tag{1}$$

The proper transient state is part (T) in Fig. 1 where the vortex shedding changes from parallel to oblique shedding with a decrease in frequency and velocity fluctuation amplitude. For the experiments presented here Eq. (1) yields with $f_{\max} = f_p$ a shedding angle of $\varphi \approx 18°$, which is in good agreement with angles reported in the literature.[12,16,17] The last part (C) in Fig. 1 represents the state of the natural vortex street with Strouhal frequency f_{St} and oblique vortex shedding. For the construction of differential equations, which are of the form

$$\frac{du_1}{dt} = u_2,$$
$$\frac{du_2}{dt} = \sum_{i,j=0}^{i+j\leq 3} a_{ij}(\tilde{x}) u_1^i u_2^j, \tag{2}$$

only parts (T) and (C) of a time series were used. In Eq. (2) $u_1(t)$ defines the velocity fluctuation amplitude of the von Kármán vortex street. Roesch, Eckelmann, and Hübler[5] could show that in the phase space representation the limit cycles of both the experimental and the simulated data are in good agreement. This indicates that the dynamics of the vortex street is well represented by the constructed differential equation. The dependence of the normalized coefficients,

$$\alpha_{ij} = [a_{ij}(\tilde{x}) - a_{ij}(4.7)]/a_{ij}(4.7), \tag{3}$$

of Eq. (2) on the distance $\tilde{x}$ from the cylinder is shown in Fig. 3. The values of the various coefficients $a_{ij}(4.7)$ are listed in Table I. Since the values of the normalized coefficients α_{ij} approach zero for $\tilde{x} > 2.5$ it can be concluded that

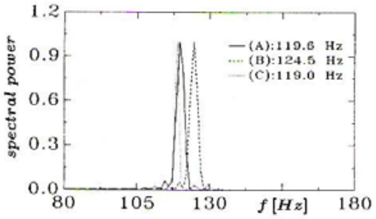

FIG. 2. Power spectra of the parts (A), (B), and (C) of the time function shown in Fig. 1. The shedding frequencies of the different parts are given in the figure.

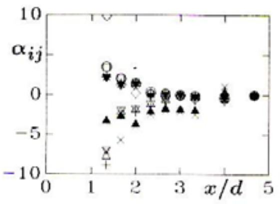

FIG. 3. Dependence of the normalized coefficients $\alpha_{ij} = [a_{ij}\tilde{x} - a_{ij}(4.7)]/a_{ij}(4.7)$ as a function of the nondimensional probe distance $\tilde{x} = x/d$ from the cylinder. Here □: a_{00}; + : a_{01}; × : a_{02}; ◊: a_{03}; ○: a_{10}; △: a_{11}; ▲: a_{12}; *: a_{20}; ▽: a_{21}; ▼: a_{30}.

TABLE I. Coefficients $a_{ij}(4.7)$ at a nondimensional probe distance $\tilde{x} = x/d = 4.7$.

Coefficient	Value: $a_{ij}(4.7)$
a_{00}	$1.18 \times 10^{+7}$
a_{01}	$7.26 \times 10^{+3}$
a_{02}	$-0.67 \times 10^{+1}$
a_{03}	-7.14×10^{-3}
a_{10}	$-2.72 \times 10^{+7}$
a_{11}	$-1.30 \times 10^{+4}$
a_{12}	$4.80 \times 10^{+1}$
a_{20}	$2.11 \times 10^{+7}$
a_{21}	$5.58 \times 10^{+3}$
a_{30}	$-5.52 \times 10^{+6}$

the dynamics of the von Kármán vortex street are developed beyond these locations. The coefficients a_{ij}, however, are dependent on the Reynolds number. Tentative measurements yield that for $110 \leqslant \mathrm{Re} \leqslant 120$ the development of these coefficients is linear.

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