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UNIQUE MODELS FOR STOCHASTIC DYNAMICS

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Abstract:

Starting from a noisy set of experimental data, we reconstruct maps in order to describe and control the dynamics of the corresponding system. When including long-time correlations and coloured noise the problem of modeling leads to the problem of finding the absolute maximum of a likelihood-function.

1. Introduction:

The dynamics of a large variety of nonlinear oscillators and a lot of other complex systems can be well described by special maps like Poincaré maps. These maps can be used for an effective prediction and control of the system [1]. When Gaussian noise is assumed, this maps can be calculated using a Least-Square-Fit or correlated methods [2]. However, colored noise cannot be modeled with this method. Furthermore it is necessary to include long-time correlations in order to get a better model [3], especially when the data are not homogeneously distributed in a certain range of state space. In this paper we calculate a likelihood-function [4]. The maximum of this function can be calculated by means of computer algebra [5] and leads to the coeffìcents of the model.

2. Reconstruction of maps from experimental time series

We start from a given set of N experimental data $\vec{x}_i^e$, $i = 1, 2, \ldots, N$, where $\vec{x}_i^e$ are n-dimensional vectors. We assume that the complex dynamics can be modeled by a low dimensional deterministic model with additional noise

$$\vec{y}_{i+1}(y_i) = \vec{F}(\vec{y}_i, \vec{p}) + \vec{F}_N(\vec{q})$$

where $\vec{p}$ and $\vec{q}$ are the parameters of the model, $\vec{F}_N$ is colored noise. For simplicity, in the following only the one-dimensional case will be considered, but a generalization for higher dimensional systems is straightforward.

In order to estimate the parameters $\vec{p}$ and $\vec{q}$ we use a maximum likelihood method. The transition propability functions are:

$$T_1(y_i, y_{i+1}; \vec{p}, \vec{q}) = D\left(y_{i+1} - F(y_i, \vec{p}), \vec{q}\right)$$

$$T_2(y_i, y_{i+2}; \vec{p}, \vec{q}) = \int T_1(y_i, y_{i+1}; \vec{p}, \vec{q}) \cdot T_1(y_{i+1}, y_{i+2}; \vec{p}, \vec{q}) dy_{i+1}$$

where T_j is the probability of transition from y_i to y_{i+j}, $\vec{p}$ are the parameters of the deterministic part of the model and $D(x, \vec{q})$ is a propability density function with $\vec{q}$ as the parameters of the color of the noise. The integrals of T should be calculated analytically, thus we propose to use polynomials as density functions. $T_3, T_4, \ldots$ are defined analogously, so that the propability of a piece of the trajectory starting at y_n leading over l points is given by $P(y_n, y_{n+1}, \ldots, y_{n+l}) = T_1 \cdot T_2 \cdots T_l$.

We use the Maximum-Likelihood-Estimation for the comparison of the probabilities of the eperimental trajectories and the trajectories predicted by the model. Thus, the optimal parameters of the model can be found by calculating the absolute maximum of

$$L = \prod_{i=1}^{N-1} \prod_{j=1}^{l} T_j(y_{i+j}^e, y_i^e; \vec{p}, \vec{q})$$

3. Example We considered a special dynamics generated by a logistic map and an approximately Gaussian noise:

$$T(y_i, y_{i+1}; a, d_f) = E_{app}\left((y_{i+1} - a y_i(1 - y_i))^2 / {d_f}^2\right)$$

with $E_{app}(x) := 0.987539 - 8.87895x^2 + 32.7768x^4 - 59.5967x^6 + 52.3996x^8 - 17.576x^{10}$

For various values of d_f the parameter a could be reconstructed from a short time series (10 or 50 data) with a precision of approx. 10%. Using 500 data it was also possible to get an estimate for d_f.

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