

# ALGORITHMS FOR THE CONSTRUCTION OF DIFFERENTIAL EQUATIONS FROM EXPERIMENTAL DATA

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Abstract: The dynamics of a Karman vortex street (KVS) in its periodic regime can be modeled by a low dimensional system of ordinary differential equations (ODE). It can be deduced from the state space representation of measured time series, that a suitable form of this ODE is an extended Van der Pol equation [1]. To obtain the coefficients of the ODE, we present a new fit method, which is able to recognize typical dynamical features of the system, e.g. stability of limit cycles, even from data with high noise level and small transients.

## 1. Introduction

Starting from fundamental equations, mode analysis and slaving principle have been used successfully to find a description of complex systems with low dimensional ODE's [2]. But up to now there has been no analytical application of these methods to the KVS. Therefore a low dimensional description has to be found from experimental data.

## 2. Construction of ODE

From an experimental time series we obtain a state space representation of the data by numerical differentiation or time delay. A lower bound for the dimension of state space is given by the condition that trajectories may not intersect [3]. A further differentiation gives flux vectors in each measured point of state space [4]. These flux vectors are fitted by a power series. The coefficients of this power series are the parameters of an expansion of the describing ODE. As a test, the gained ODE is numerically integrated and compared with the original data [5] [6].

## 3. Fit method

On the limit cycle the flux vectors do not contain any information about stability. In its vicinity this information is destroyed by noise. As it is difficult to obtain data far from the limit cycle, we use a fit which takes into account long time dynamics. A preliminary ODE of the form

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \sum_{i,j=0}^{3,3} a_{ij} x_1^i x_2^j\end{aligned}$$

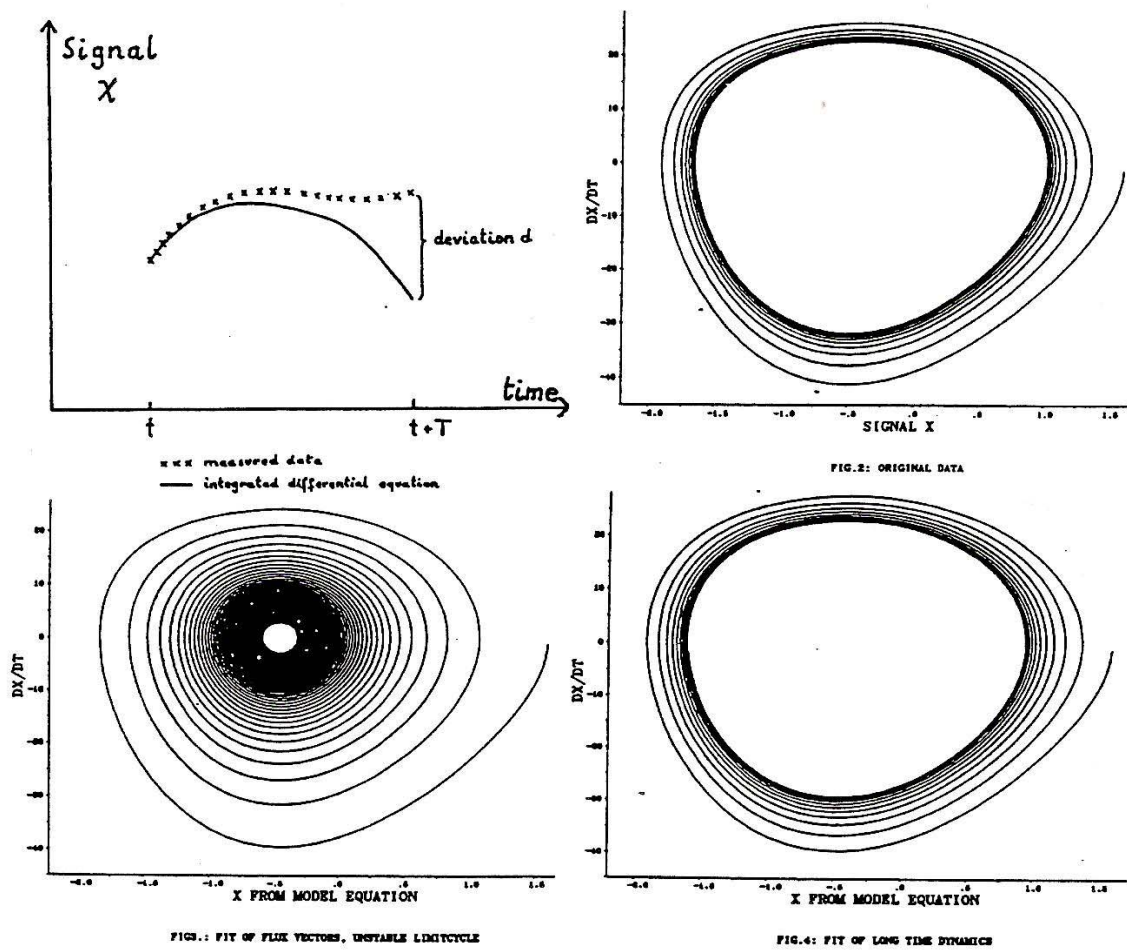
– a result of the conventional fit – is numerically integrated, starting from several measured points  $x(t)$  of the time series. After the integration time  $T$ , integration arrives at  $x'(t+T)$ , the time series shows  $x(t+T)$ . The difference  $d = x' - x$  is minimized by iteratively

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changing the parameters  $a_{ij}$ . The principle is illustrated in fig. 1. Fig. 3 and 4 show the difference between conventional and long time fit, applied to data with small transients (fig.2).

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