

RESONANT STIMULATION AND CONTROL OF KARMAN'S  
VORTEX STREET

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**Abstract:** We show that the periodic dynamics of a vortex street behind a circular cylinder can be modeled by a low dimensional system of ordinary differential equations with a few parameters, although the system has infinitely many degrees of freedom. The parameters depend on the hydrodynamic control parameters. We show experimentally that the dynamics is mainly on an inertial manifold. All degrees of freedom destined away from the inertial manifold are slaved. Furthermore we show, that the system can be controlled by nonsinusoidal acoustic perturbations. The system remains on the inertial manifold if the perturbations are resonant.

1. Introduction

During the past several years there has been theoretical progress in hydrodynamics. Effinger and Großmann/1/ have calculated analytically the Kolmogorow spectrum for turbulent systems by a mean field theory. Furthermore, a huge variety of experiments shows, that there is an excellent agreement between theory and experiment for stationary and periodic systems/2,3/. In 1963 Lorenz/4/ pointed out that in the parameter region between periodic and turbulent state there might exist systems which can be modeled by strange attractors and low dimensional ordinary differential equations. Liapounov exponent and generalized dimensions indicate that there are low dimensional strange attractors in the Taylor-Couette system/5/, Benard system/6/, and the vortex shedding from a circular cylinder/7/. Recently a method has been presented for a reconstruction of nonlinear differential equations from experimental data/8/. The aim of this paper is to show that it is possible to reconstruct a differential equation from the dynamics of the flow of a vortex system behind a circular cylinder. Furthermore we show that the flow can be controlled/9/ in a very efficient way by using these differential equations.

## 2. Experimental results

The measurements have been done in the jet region of a wind tunnel which is similar to the wind tunnel of Eckelmann and Roesch/8/. The circular cylinder used has a diameter of 2 mm (steel, length=250mm) and is located 50 mm behind the outlet of the tunnel perpendicular to the jet. All velocity signals were obtained with a hot wire operated on a constant temperature anemometer. For nearly all signals, digitation was done on a sufficiently high frequency (10 kHz or more). The reconstruction of differential equations was done with the algorithm of Cremers et al. /10/. Smoothing and differentiating of the data were performed by a standard spline interpolation. The resulting flow vector field was approximated by a series of Legendre polynoms.

Fig. 1a shows a limit cycle drawn from the dynamics of the hot wire probe (Reynolds number  $Re=70$ ). The position of the hot wire probe is in the center of the vortex street 6mm behind the center of the cylinder. 2000 data points which represent about 30 oscillations have been used to reconstruct an ordinary differential equation of the type

$$\ddot{v} = \sum_{i=0, j=0}^{3,3} a_{i,j} v^i \dot{v}^j \quad (1)$$

where  $a_{i,j}$  are parameters. We estimated these parameters by a least square fit of the flow vector field resulting from the experimental data.

Fig 1.b shows the dynamics of the resulting differential equation. We found an excellent agreement between the geometry of the limit cycle of the measured data and the numerical simulation for various  $Re$  numbers ( $0 < Re < 80$ ). Since most of the parameters vary strongly, may be due to experimental noise, we could not find a simple relation between the parameters of the differential equation and the Reynold number.

Further we investigated the response of the vortex system. We applied a sinusoidal

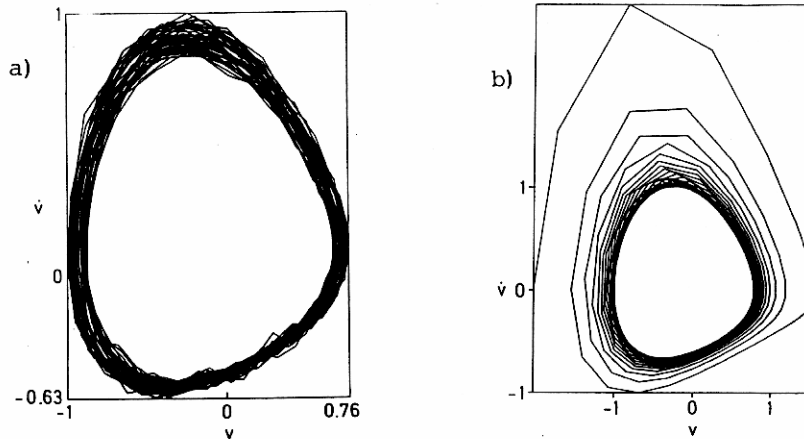


Fig.1 State space representation of the velocity signal  $v$ .  $v$  and  $\dot{v}$  are normalized to their maximum value. (a) shows the experimental data and (b) a numerical simulation of the estimated differential equation.

and a square wave acoustic perturbation perpendicular to the flow by two loudspeakers. Fig.2a shows that the mode region of entrainment of a square wave response is essentially larger than the region of entrainment for sinusoidal perturbations. Fig. 2b shows the response of the vortex system to a very low frequency excitation. After the perturbation the dynamics is composed by a very low frequency motion and the usual quick oscillations. The low frequency motion is slowly damped out.

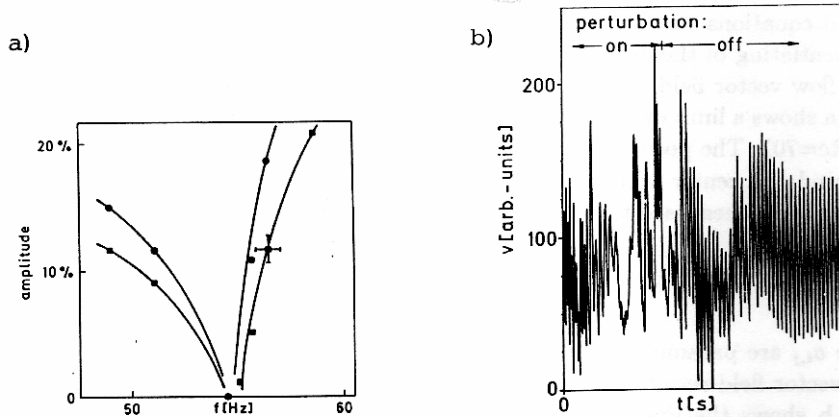


Fig.2a The boundaries of the region of entrainment for sinusoidal (●) and for square wave perturbations (■). The position of the hotwire is 6mm behind the cylinder in the center of the vortex street. The amplitude (distance between the extrema) of the acoustic perturbation is normalized to the amplitude of the variation of the velocity of the unperturbed system at this position. Fig.2b The response to a low frequency perturbation ( $f = 0.1\text{Hz}$ ) versus time.

### 3. Discussion

The experimental results indicate that the dynamics of the velocity signal of the vortex street can be modeled by one dimensional oscillators with a nonlinearity up to third order since there is an excellent agreement between the limit cycle of the experiment and the limit cycle of the simulation. Fig. 1a indicates that the divergence of nearby trajectories is positive for  $v > 0$  and negative else. Recently, it has been shown by nonlinear control theory that the region of entrainment of Van-der-Pol oscillators can be increased by a square wave perturbation/11/. The reason for the enlargement is that the divergence of nearby trajectories of the Van-der-Pol oscillator just depends on the amplitude  $x(t)$  and is independent of the velocity  $\dot{x}$ . Since the differential equation of a Van-der-Pol oscillator and Eq.(1) are closely related and the divergence of both systems is nearly independent of the dotted variable this property might be the reason for the enlargement of the region of entrainment. From this discussion and from the successful

reconstruction of a differential equation from the experimental data we conclude that the dynamics of the velocity signal of the vortex street can be modeled by a low dimensional system of differential equations. Of course there are other excitations which are not included in such a simple model, as Fig. 2b indicates. The state space of hydrodynamic systems has an infinite number of degrees of freedom. The dynamics of the variables of the model is in a subspace, which is called inertial manifold. Fig. 2b indicates that it is possible to stimulate some degrees of freedom destinating away from the inertial manifold but it indicates further that these excitations are damped out after a while. We conclude that there is a large variety of degrees of freedom destinating away from the inertial manifold which are slaved/2/, which are not explicitly included in the model, but which can be excited by strong perturbations. However a general feature of the response of most real systems is, that the excitation is small if the perturbation is small. If the goal of the perturbation is to shift the basic frequency of the vortex street, the stimulation with square waves has two advantages: first, due to the enlargement of the region of entrainment it is possible to shift the frequency of the experimental system by a perturbation of small amplitude, and second, one can predict the response using a low dimensional system of differential equations since the excitations destinating away from the inertial manifold generally remain small. Generally nonlinear control theory /9/ provides us with the possibility to calculate perturbations which satisfy a certain condition or goal, but are of small amplitude. Usually these *resonant* perturbations are aperiodic but the amplitude of excitations destinating away from the subspace of the model remains small. May be perturbations of this type can be used to control chaotic flows.

We like to acknowledge H. Haken and O. Wohofsky for continuous support and W. Eberl for computer programs. Furthermore we acknowledge fruitful discussions with H. Eckelmann, E. Roesch, and R. Friedrich.

- /1/ H. Effinger, and S. Grossmann, Z. f. Phys. B 66, 289(1987)
- /2/ H. Haken, Advanced Synergetics (Springer, Berlin 1983)
- /3/ J. Zierep, and H. Oertel (eds) Convective Transport and Instability Phenomena (Braun, Karlsruhe 1982)
- /4/ E. Lorenz, J. Atmospheric Sci. 20, 130 (1963)
- /5/ P. Fenstermacher, H. Swinney, J. Gollup, J. Fluid Mech. 94, 103 (1979)
- /6/ M. Jensen, L. Kadanoff, A. Libchaber, I. Procaccia, J. Stavans Phys. Rev. Let. 55, 2798 (1985)
- /7/ D. Olinger, K. Sreenivasan, Phys. Rev. Let. 60, 797 (1988)
- /8/ E. Roesch, Rekonstruktion von Differentialgleichungen aus experimentellen Daten der Kármánschen Wirbelstraße Diplomarbeit Max-Planck-Institut für Strömungsforschung Göttingen (1988)
- /9/ A. Hübler, Beschreibung und Steuerung nichtlinearer Systeme, Dissertation Techn. Universität München (1987)
- /10/ J. Cremers, A. Hübler, Z. Naturforsch. 42a, 797 (1987)
- /11/ A. Hübler et al., preprint 1988

+ part of Ph.D. thesis