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# ADAPTIVE CONTROL OF CHAOTIC SYSTEMS

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Abstract: In order to describe the chaotic dynamics of a nonlinear system, a discrete map is reconstructed from the time series of an experimental system. The parameters of the map may depend on time. The map is a model for the dynamics of the experimental system. This model can be used in order to control the dynamics of the experimental system with small external perturbations, e.g. in order to get a special periodic dynamics or a special type of chaos. We argue that modelling and controlling can be done simultaneously.

## 1. Introduction

For a large variety of nonlinear oscillators higher order Fourier amplitudes fall off rapidly/1/. The dynamics of these oscillators can be aproximated with high accurance by a smooth interpolation between the extrema of the exact dynamics. The extrema of the dynamics can be calculated with special Poincare maps. The time between the extrema is the recurrence time of the Poincare map. Recently it has been shown, that these maps can be extracted from the time series of an experimental system/2,3,4/. Further it has been shown numerically/5/ and experimentally /6/, that these maps can be used for a control of the nonlinear system. In this paper we show that modelling and controlling can be done simultaneously.

## 2. Reconstruction of a map from an experimental time series

We assume, that N experimental data $\mathbf{x}_i^e$, where $i = 1, 2, \ldots, N$ are continuously distributed in a certain range of the state space. By an appropriate rescaling of the coordinates of the state space $\mathbf{x}_i^e \rightarrow \mathbf{y}_i^e = \mathbf{S}(\mathbf{x}_i^e)$, where $\mathbf{x}_i^e$ are n-dimensional vectors and $\mathbf{S}$ is an n-dimensional function the data can be homogeneously distributed in the unity cell of state space. For simplicity we restrict the following investigations to $n = 1$, but a generalisation to higher dimensional systems is straightforward. We assume that the experimental time series can be modelled by a map of type

$$y_{i+1}(y_i) = \sum_{k=1}^{\infty} b_k \sin(\pi k y_i) + F\left(\sigma_F^2, i\right) + F_c(i) + b_0 + b_0' y_i \tag{1}$$

where $y_i$ are one dimensional rescaled state variables, $i$ represents time, $b_k$ are the parameters of the model, $b_0 = y_{i+1}(0)$, $b_0' = y_{i+1}(1) - y_{i+1}(0)$, and $F$ is a uncorrelated random force. $\sigma_F^2$ is the variance of $F$. $F_c$ represents perturbations which are applied onto the system in order to control it. We extract the parameters of the values $b_k^e$ of the parameters $b_k$ from the experimental data by a maximum likelihood estimation/6/:

$$b_k^e = \frac{2}{N} \sum_{i=1}^{N} \left(y_{i+1}^e - F_c(i) - b_0 - b_0' y_i^e\right) \sin(\pi k y_i^e) \tag{2}$$

where the variance of $b_k^e$ is

$$\sigma_{b_k}^2 = \frac{16\sigma_F^2}{\pi^2 N} \tag{3}$$

In order to investigate the quality of the model we use a F-test /7/. The model is accepted if the following relations hold:

$$Q = \frac{\sigma_m^2}{\sigma_F^2} = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(y_{i+1}(y_i^e) + F_c(i) + b_0 + b_0' y_i^e - y_{i+1}^e\right)^2}{\sigma_F^2} < f_{\alpha,N} \tag{4}$$

and

$$Q^{-1} < f_{\alpha,N} \tag{5}$$

where $f$ is the $f$-distribution function /7/ and $\alpha$ is the confidence level. Now we rewrite the left side of Eq.(4)

$$\sigma_m^2 = \frac{1}{N} \sum_{i=1}^{N} \left(y_{i+1}^e - F_c(i)\, b_0 - b_0' y_i^e\right)^2 - \frac{1}{2}\left((b_1^e)^2 + (b_2^e)^2 + \ldots\right) \tag{6}$$

Eq.(6) illustrates that Eq.(4) generally remains valid if some small parameters are set to zero. The decision which of these small parameters can be neglected depends on further information on the system or can be done by using more data. It can be easily shown, that the values $b_k^e$ are independent of $F_c$.

3. Control of chaotic systems

In order to control the system we define an goal dynamics

$$w_{i+1} = G(w_i) \tag{7}$$

where $w_n$ represents a state of the experimental system, and G is a real function. The goal of the control is to get $y_i = w_i$. This can be done by $F_c(i) = -\sum_{k=1}^{\infty} b_k \sin(\pi k w_i) - b_0 - b_0' w_i + G(w_i)$, if $y_i = w_i$ is a stable solution of Eq.(1)/8/.

4. Numerical example

In order to investigate the control of a chaotic system we use a logistic map/9/

$$x_i^e = a x_i^e (1 - x_i^e) + F\left(\sigma_F^2, i\right) + F_c(i) \tag{8}$$

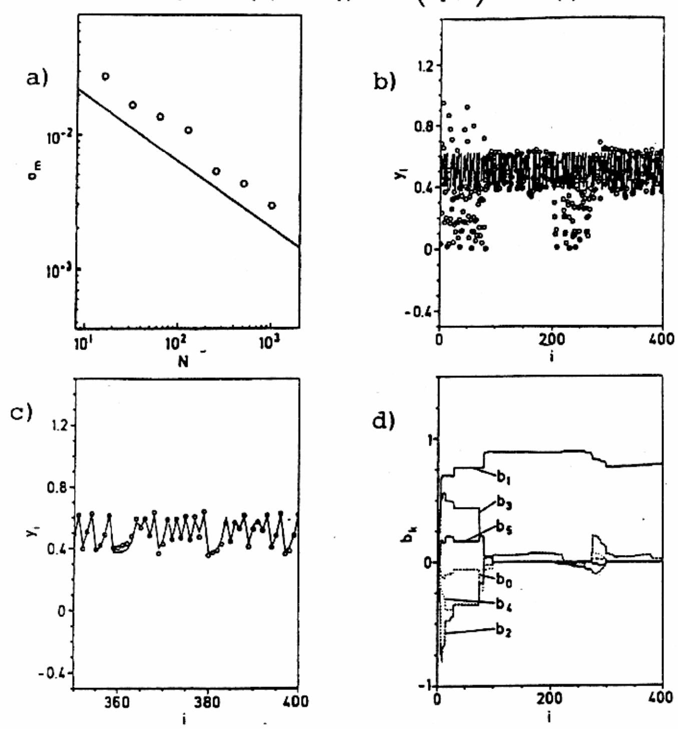


Fig. 1 Control of the logistic system: Fig. 1a the numerical (o) and the theoretical (-) value of $\sigma_{b_k}$ versus N ($\sigma_F = .05$), Fig.1b,c the dynamics of the controled system (o) and the goal dynamics (-) versus time($\sigma_F = .01, N = 100$, b=4), Fig. 1.d the parameters of the model versus time, where $S = 4(x_i^e - .375)$

and the following goal equation

$$\begin{aligned} w_{i+1} &= 4(z_{i+1} - .375) \\ z_{i+1} &= bz_i(1 - z_i) \end{aligned} \tag{9}$$

Eq. (9) shows that the aim dynamics is closely related to the dynamics of a logistic map. For $b = 4.$ the goal dynamics is chaotic.

Fig. 1a illustrates the relation between the variance of the estimated parameters and the number of data N. There is a good agreement with Eq. (3). The parameters of the model are extracted from the data in the range from $i - N$ to $i$. Fig. 1b shows the control of the logistic system Eq.(8). For $0 < i < 200$ $a$ is set to 3.8 and the dynamics of the unperturbed system would be chaotic. Afterwards the unperturbed system would have a periodic dynamics, since $a$ is set to 3.3. Fig. 1b and Fig.1c illustrate that it is not possible to control the logistic system directly after a sudden change of the parameters. But after a delay which is approximately equal to the number of data N (Fig. 1d) a control is possible again (Fig.1c). Further investigations show that no control of the dynamics is possible if N is too small, probably due to the uncertainty of the parameters of the model (see Eq.(3)).

A generalisation of these methods for systems which can only be modelled by differential equations is straightforward.

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