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RESONANT STIMULATION AND CONTROL OF NONLINEAR MECHANICAL PENDULUM BY POINCARÉ MAPS

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Abstract: A new method for resonant stimulation of nonlinear damped oscillators by nonlinear entrainment is presented. Appropriate driving forces are calculated with Poincaré maps. These maps can be extracted from experimental time series. We show experimentally, that the resonant driving forces are in phase with the velocity of the oscillator and cause a huge energy transfer. The corresponding driving forces are aperiodic and can be calculated without any feed back from the experiment.

1. Introduction

It has been shown numerically/1/ and experimentally/2/ that a nonlinear damped oscillator can be stimulated resonantly by nonlinear entrainment. For a large variety of nonlinear oscillators higher order Fourier amplitudes fall off rapidly/3/. The dynamics of these oscillators can be approximated by a smooth interpolation between the extrema of the exact dynamics. The extrema of the exact dynamics can be calculated with special Poincaré maps. The time between the extrema is the recurrence time of the Poincaré map. Since resonant driving forces are closely related to the dynamics of the unperturbed system they can be estimated from these Poincaré maps too/4/. The aim of this paper is to show experimentally that a resonant stimulation of nonlinear oscillators is possible, even if the resonant driving force is estimated from a Poincaré map.

2. The experimental setup

As a real physical nonlinear oscillator we used a damped wheel with a excentric mass distribution/5,6/. The dynamics of the pendulum can be modelled by

$$\Theta \cdot \ddot{y} + \eta \cdot \dot{y} + c_1 \cdot y - c_2 \cdot \sin(y) = F(t) \qquad (1)$$

where y is the angular displacement, $\Theta = 1.65 \cdot 10^{-3}$ kgm^2, $c_1 = 1.62 \cdot 10^{-2}$ kgm^2 s^{-2}, $c_2 = 0.03563$ kgm^{2}s^{-2} and η the friction constant which can be varied in the range of $5 \cdot 10^{-5}$ kgm^{2}s^{-1} $< \eta <$ $6 \cdot 10^{-4}$ kgm^{2}s^{-1}.The time dependent driving force F(t) is transmitted by a digital-to-force converter from a computer to the experimental pendulum.

3. Estimation of the driving force

3.1. Numerical estimation of the resonant driving forces

It has been shown / 1,2 / that driving forces F(t) are resonant if $F(t) = (1-\alpha) \cdot \eta \cdot \dot{x}$ where α is a real constant and where the dynamics of x is given by

$$\Theta \cdot \ddot{x} + \alpha \cdot \eta \cdot \dot{x} + c_1 \cdot x - c_2 \cdot \sin(x) = 0 \qquad (2)$$

and where x(0) approx. y(0). We integrate Eq. (2) numerically with a Runge-Kutta algorithm of 5.th - 6.th order and approximate the next-extrema map and the recurrence time between the extrema by a polynom of fifth order.

$$P_y: \quad \dot{a}_{n+1} = \sum_{i=0}^{5} c_{n,i} \cdot (\dot{a}_n)^i \qquad T_n : T_n = \sum_{i=0}^{5} c_{n,i} \cdot (\dot{a}_n)^i \qquad (3)$$

where $\dot{a}_n$ is the *n*-th extrema , of the dynamics of $1.05 \cdot (1-\alpha) \cdot \eta \cdot \dot{x}$ and T_n is the time between the $\dot{a}_n$ and $\dot{a}_{n+1}$. F(t) results from a smooth spline interpolation between the data resulting from an iteration of Eq.(3). We took initial values for the iteration which are close to the stationary state of the unperturbed system.

3.2 Estimation of the resonant driving forces from the Poincaré map of the unperturbed system

For $\alpha = -1$ Eq. (2) results from Eq. (1) by a reflection of time. In this case the dynamics of $\dot{a}_n$ can be calculated from the dynamics of the extrema of $\dot{x}$ the unperturbed system by a reflection of time. F(t) can be estimated by a smooth interpolation of the corresponding Poincaré map Eq. (3).

4. Experimental results

Fig. 1b shows the resonant stimulation of the pendulum ($\eta = 6\cdot10^{-4}$ kgm^{2}s^{-1}, $\alpha = -.17$) where the Poincaré map was calculated numerically. In Fig. 1a we depict the response of the experimental pendulum for $\eta = 3\cdot10^{-4}$ kgm^2 s^{-1} and $\alpha = -1$ where F(t) was estimated from the dynamics of the unperturbed systems. Due to the strong amplitude frequency coupling /7/ of the pendulum there is a sensitive dependance of the basic

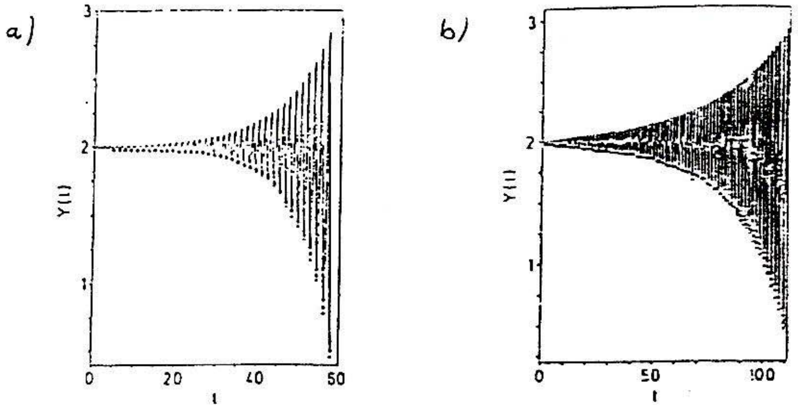

Fig. 1 The extrema of the amplitude y of the experimental pendulum versus t for $\alpha = -1$ (a) and $\alpha = -0.17$ (b). The continuous line represents the theoretical dynamics of y.

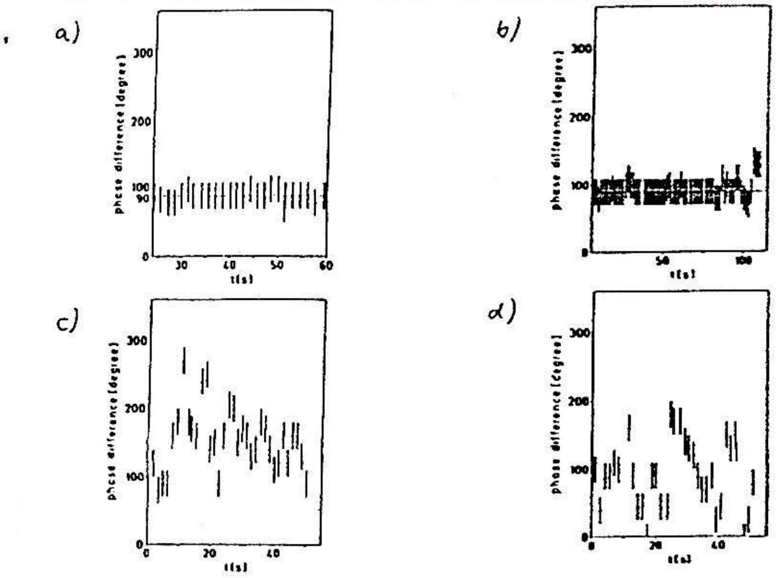

Fig.2 The difference in phase between F and y versus t. (a,b): Resonant stimulation by Poincaré-Map, (a): $\alpha=-1$, $\eta=3\cdot10^{-4}$, (b): $\alpha=-.17$, $\eta=6\cdot10^{-4}$; (c,d): Sinusoidal stimulation with $F=4\cdot\sin(\omega_e\cdot t)$, ω_e: eigen frequency in the linear region , (c): $\eta=3\cdot10^{-4}$, (d): $\eta=6\cdot10^{-4}$.

frequency of the oscillator from the amplitude of the oscillation. Fig. 2 illustrates that the phase relation between the driving force and y remains 90 degree, despite of the large shift in frequency. The phaseshift of 90 degree between y and F indicates that the reaction power is zero and that the driving force is resonant/1/. If the driving force is sinusoidal this resonance condition is not satisfied (Fig. 2c,d).

5. Conclusions

We have shown that nonlinear oscillators can be stimulated resonantly by driving forces which are estimated from Poincaré maps. These Poincaré maps can be estimated analytically/8/, numerically or directly from the unperturbed system. Therefore the stimulation can be done without any idea on the differential equation of the system. No direct feed back from experimental system is necessary during the stimulation. If the Poincaré map has been estimated, the whole time dependence of the driving forces can be calculated before the stimulation starts.

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References

/1/ G. Reiser, A. Hübler, E. Lüscher, Z.f.Naturforschung 42a, 803(1987)

/2/ C. Wagner, W. Stelzel, A. Hübler, E. Lüscher, Helv.Phys.Acta 61, 228(1988)

/3/ A.J. Lichtenberg, M.A. Lieberman, Regular and Stochastic Motion (Springer, Berlin 1983) p.103

/4/ A. Hübler, R. Georgii, M. Kuchler, W. Stelzel, E. Lüscher, Helv.Phys.Acta 61, 897(1988)

/5/ G. Mayer-Kress, Zur Persistenz von Chaos und Ordnung, Ph.D. Thesis, Inst. für Theoretische Physik und Synergetik, Universitaet Stuttgart 1984

/6/ S.Beckert, U.Schock C.D.Schultz,T.Weidlich, F. Kaiser, Phys. Letters 107A, 347 (1985)

/7/ A.H. Nayfeh, D.T. Mook, Nonlinear Oscillations, (John Wiley & sons, New York 1979) p.54

/8/ W. Eberl, M. Kuchler, A. Hübler, E. Lüscher, Z.f. Physik B68, 253(1987), R. Wackerbauer, W.Eberl, A.Hübler, E.Lüscher, Helv.Phys.Acta 61, 236(1988)

/9/ M. Kuchler, W. Eberl, A. Hübler, E. Lüscher, Helv.Phys.Acta 61, 232(1988)