Helvetica Physica Acta
Vol. 62 (1989)

0018-0238/89/030282-04$1.50+0.20/0

PROPERTIES OF HYDRODYNAMIC SYSTEMS WITH FLEXIBLE BOUNDARY CONDITIONS

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Abstract: *In a circular channel we investigate experimentally the dynamics of a solid body which can move perpendicular to the axis of the channel. If the flow in the tube is slow and steady the stable stationary state of the system satisfies a variational principle. We show that at this state the Reynolds number reaches its maximum value. A generalisation of these results for bodies with deformable boundary conditions might have important applications in technical hydrodynamics.*

1. Introduction

It has been proved by Helmholtz and Korteweg /1/ that if the velocities at the boundary are given, the slow steady motion of an incompressible viscous liquid satisfies the condition of making the dissipation an absolute minimum. Feynman /2/ has shown, that the dissipation due to the Ohmic resistance reaches its minimum value at the steady state if the boundary conditions for the current are kept fixed. Recently it has been shown /3,4,5/ that the dissipation reaches a minimum at the stationary state too if the boundary conditions for the current vary essentially slower than the typical relaxation time of the distribution of the electric carriers. In this paper we investigate analogous hydrodynamic systems where the boundary conditions are not kept fixed but may change slowly compared with the typical time scales of the flow.

2. Experimental results

A solid body (Fig. 1) hangs on a thin wire (Φ = 0.1 mm, length = 45 cm) in a vertical circular channel (Fig.2). The fluid is oil. With two valves the amount of oil *Q* flowing through the channel is regulated. If the level of the oil in the channel stays on a constant value Q is nearly independent of the position of the solid body.

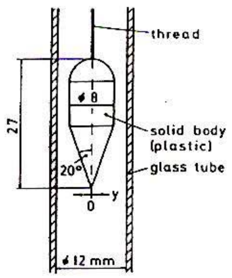

Fig. 1: Streamline shaped body (weight: 1.28 g) in a circular channel

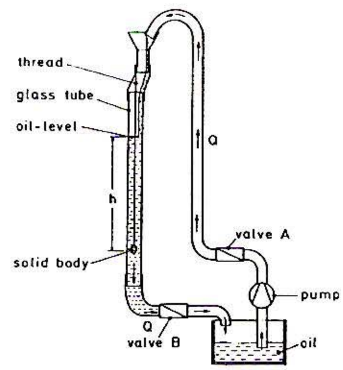

Fig. 2: Circular channel with a solid body

We investigate experimentally the stationary position y of the solid body for several values of Q. Fig. 3 indicates that the solid body approaches the boundary of the channel at low values of Q . At larger values of Q a position in the center of the channel seems to be stable.

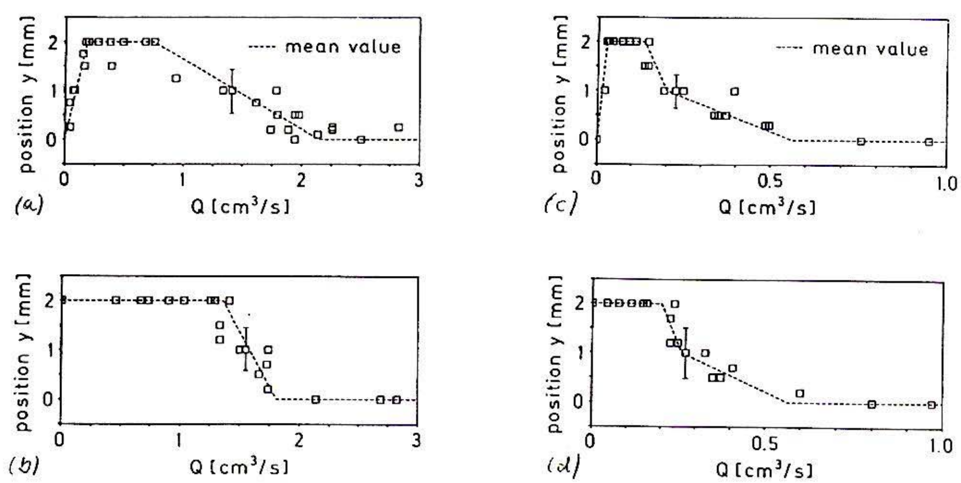

Fig. 3: The distance y of the center of the solid body from the center of the channel versus Q for two different liquids. Liquid A: viscosisty η = 0.045 Pa s, density ρ = 0.86 g/cm^3 (a,b) ; Liquid B: η = 0.32 Pa s, ρ = 0.87 g/cm^3 (c,d) . The stationary state for Q = 0 is at y_o = 0 (a,c) and at y_o = 2 mm (b,d).

3. Discussion

In order to explain the experimental results we use a simplified model. The outflow of an reservoir is divided into two channels by a plate which can move perpendicular to the axis of the channels (Fig. 4).

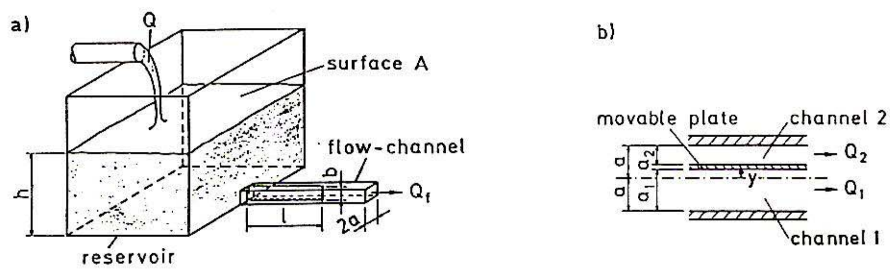

Fig. 4: A movable plate divides the outflow of a reservoir into two channels. The pressure at the end of the outflow is zero, *Q* is kept at a fixed value.

If the dynamics of the incompressible, viscous flow in the channels is estimated by the relations of a Poiseuille flow/6/ and the relaxation time of the motion of the fluid is essentially smaller than the typical time scale of the dynamics of the plate the potential energy can be estimated by:

$$V = c \cdot \frac{\eta^2 Q^2}{\rho} \cdot \frac{1}{(a^2 + 3y^2)^2} \qquad (1)$$

where $c = 18 A l^2 / (g a^2 b^2)$ with the geometrical quantities A, l, a, b (see Fig. 4), gravitational constant g, viscosity η, density ρ and $|y| \leq a$.
In this case the motion of the fluid is slaved /7/ by the position of the plate. Therefore there is no explicit time dependence of V. If we assume, that V reaches a minimum at the stable stationary state we have $y = \pm a$ for stable stationary states, i.e. one channel is closed. In this case the dissipation

$$P = \frac{6l}{ab} \cdot \eta Q^2 \cdot \frac{1}{(a^2 + 3y^2)} \qquad (2)$$

reaches its minimum value too. In this simple system the dynamics of the plate models the dynamics of the solid body, the reservoir models the storage of the fluid in the real channel. The simple model seems to explain the stabilisation of the position at the boundary of the channel. Since the motion of the solid body is not exactly horizontal there is a small additional gravitational force acting into the direction of the position of the solid body for $Q = 0$. Therefore the position at the boundary becomes unstable for small Q if $| y_0 | \neq a$. Up to now we have no explanation for the destabilisation of $| y | = a$ for large values of Q.

4. Conclusion

If gravitational forces can be neglected or if the solid body moves exactly horizontal, the solid body approaches a stable state at the boundary for small values of Q . At this state the Reynolds number reaches its maximum value and the dissipation its minimum value. The simple model indicates that a body with a deformable shape, e.g. a flexible cover with a medium of very high viscosity inside should approach a state with the same properties if the time scale of the deformation is essentially longer than the time scale of the fluid in the channel. An application of this effect might be the improvement of streamline shaped bodies.

We acknowledge H. Haken and O. Wohofsky for continuous support.

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