Extremal Properties of Dendritic Patterns : Biological applications II

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Abstract

Stationary dendritic patterns, as e.g. stable agglomerates of metallic spheres in an electric field, exhibit extremal properties. Biological systems, like root systems of plants, show dendritic structures. A biological potential for such biological systems is defined. Experiments show that this biological potential is minimal in the stationary state.

Introduction:

The geometry of optimal dendritic patterns in an open physical system, like the mentioned metallic-ball-system, satisfies a variational principle /2/. Recently it has been shown that the potential energy W_b of the system is minimal in the stationary state /2/. Dendritic geometry is the basic geometry of roots, ramified nerves, arteries, lymphvessels, etc. A theoretical model for optimal structures of roots compares the properties of stationary dendritic patterns in a physical system with stationary biological dendritic patterns /1/. The aim of this paper is to show experimental results and compare them with our theoretical model.

Theoretical model:

The purpose of the root system of plants is the mechanical support of the plant as well as the absorption of ions and water. In our theoretical model we have shown that in the stationary state the biological potential (potential energy) W_b satisfies the scaling relation

$$W_b \sim M^{-D_M} \sim N^{-D_N} . \tag{1}$$

M is the number of tips and N represents the total mass of the root system. D_M and D_N are factors depending on the geometry of the experiment, as well as on the ion current traversing the system, which are independent of M and N, respectively /1/.

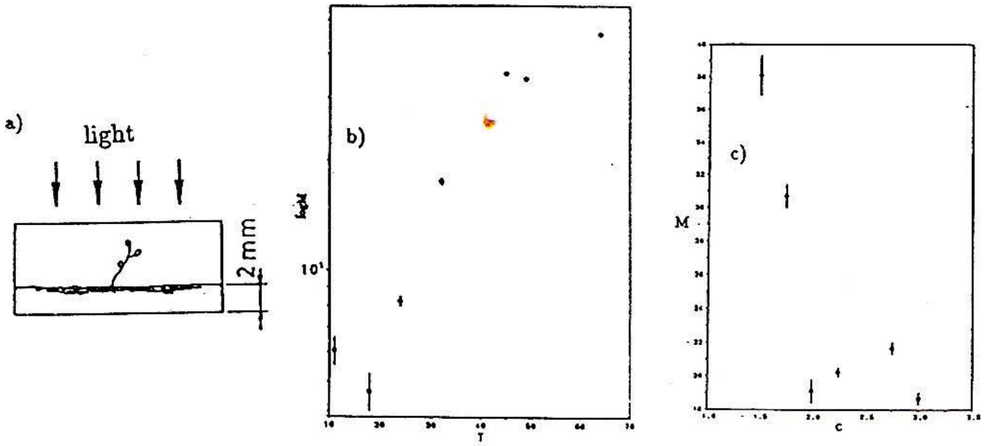

Figure 1: a) Experimental set-up, b) Number of tips M as a function of time T, c) Number of tips M as a function of the control parameter c

Experiment and results:

Seeds of sinapis arvensis are grown in a 2mm layer of 100 g soil within a flat container. The root system thus develops nearly two-dimensionally. For analysis the number of tips M is counted and the root structure is digitized. This data is fed to a computer to determine the fractal dimension DF and the biological potential W_b. The control parameter of this experiment is c, defined as the ratio of mass of water to the mass of soil in the container. When c is kept constant, after some time the system reaches a stationary state (fig. 1b)). The number of tips M and the degree of ramification (indicated by the fractal dimension DF decreases with increasing c, i.e. with more water and thus an increased number and mobility of nutritious ions (see figs 1c), 2a)). This means that the plant will adapt to the conditions in the ground with minimal expense of material. Figs 2b), 2c) show, how the biological potential W_b in the stationary state varies with c and the fractal dimension DF.

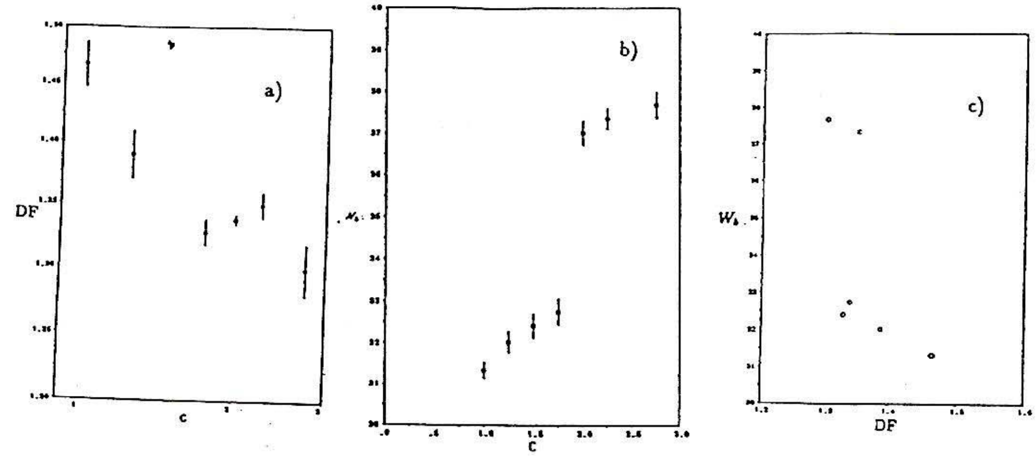

Figure 2: a) Fractal dimension DF of the root system as a function of c, b) Biological potential W_b as a function of c, c) W_b as a function of the fractal dimension DF of the root system

Discussion:

The first experimental results confirm our theory insofar as there is a direct relation between the structure of the root system (number of tips M, fractal dimension D_F) and the biological potential energy W_b, on one side, and the control parameter c on the other. Biological systems which show dendritic patterns try to adapt to a changing environment in such a way that their structure fit optimally to the demands of the system. For a given environmental condition optimal dendritic structures assure minimal expense of material by maximal coupling to the surrounding medium.

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