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CONTROL OF THE DYNAMICS OF A WATERJET

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Abstract: A method for controlling nonlinear systems by entrainment is presented. The method is used to control the geometry of a jet of water in earth's gravity field.

1. Introduction

The basic question is, whether there is a relation between turbulences and chaos. For both, the Benard and the Taylor experiment, a strange attractor has been found /1/. This may be explained by the high feedback existing inside a closed container; due to this fact it is possible to describe the dynamics by a nonlinear oscillator. In contrast for a jet or a Karman's vortex path this feedback is very low, because in these cases we have to deal with open containers. Thus no description by an oscillator seems possible, and therefore no chaos would be expected despite the claim of Chrutchfield et al /2/. The subject of this paper is to resolve the contradiction between experiment and theoretical prediction.

2. Description of the Experiment

The vertical motion of a vertical, round glass pipe (radius 3.3mm) is computer-controlled; by this motion the velocity of outflow of a waterjet u may be modulated. $u(\tau) = u_o + \varepsilon(\tau)$, where τ is the time.

3. Experimental results

The jet has been photographed for different modulations $\varepsilon(\tau)$ of the outflow velocity (fig.1).

For calculating the jet dynamics a theory analogous to that of time of flow /(3,4)/ has been used. Except gravity and inertial forces all other forces have been neglected. Thus the Euler equation and the continuity equation are obtained in the following form

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = g \, , \qquad \frac{\partial r}{\partial t} + v\frac{\partial r}{\partial z} + 2r\frac{\partial v}{\partial z} = 0 \qquad (1)$$

r represents the radius of the jet, measured in units of the radius of the outflow, v the streaming velocity, z the axis of the jet, t the time and g the gravitational acceleration on the earth's surface.
(1) are transformed into Lagrange-coordinates ν and τ:

$$t(\nu,\tau) = \nu \, , \qquad z(\nu,\tau) = u(\tau)(\nu-\tau) + \frac{g}{2}(\nu-\tau)^2 \qquad (2)$$

That means that a fluid particle, starting at time τ with velocity $u(\tau)$, is falling with constant acceleration g.
After the coordinate transformation equations (1) are

$$\frac{\partial v}{\partial \nu} = g \qquad (3)$$

$$\frac{\partial r}{\partial \nu} = k(\nu)\cdot r \qquad (4)$$

where $$k(\nu) = \frac{1}{2}\,\frac{u'(\tau)}{(u'(\tau)-g)(\nu-\tau) - u(\tau)}$$

(4) is the typical differential equation for an amplifier. Therefore the jet may be interpreted as an amplifier of the outflow velocity modulations. If the outflow velocity $u(\tau)$ is modulated only by noise, it acts as a noise amplifier. If $u(\tau)$ is deliberately modulated, essentially this modulation is amplified. In order to verify the approximation of this theory, the experiment mentioned above has been accomplished to compare the experimental results with the theoretical calculations Fig. 1 shows that good agreement between experiment and theory is achieved. Furthermore the distance between drop number i and drop number i-1 has been measured, the drops being counted from above. Fig. 2 shows the result in comparison to theory.

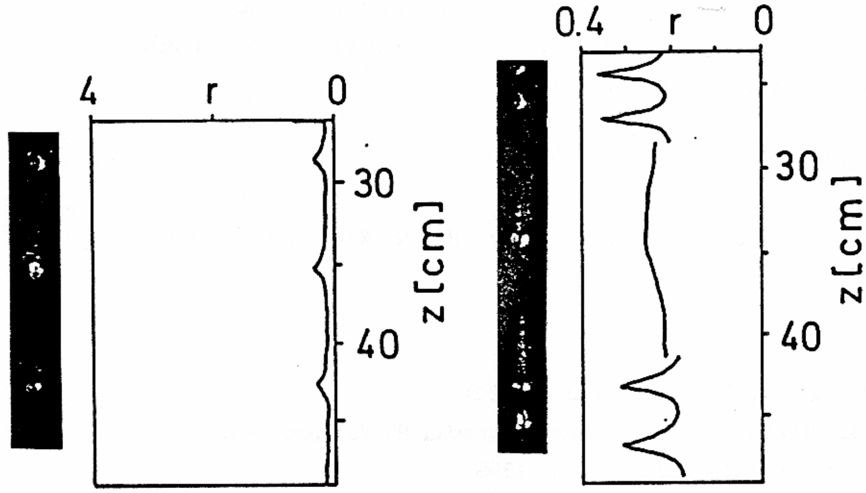

fig. 1 Comparison of the geometry of the jet (photographed) for different modulations $\varepsilon(\tau)$ with r(z,t) calculated with (4)

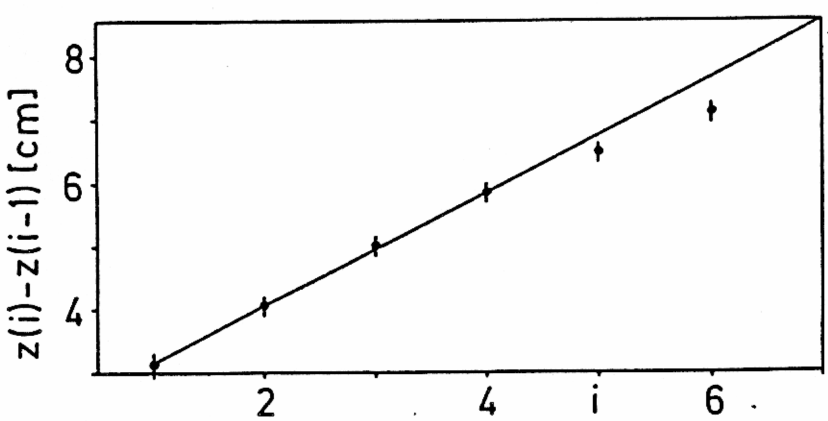

fig. 2 Distance between the drops as a function of i (ϕ experiment, —— theory)

3. Conclusion

It has been shown that the jet dynamics acts as an amplifier for the velocity modulations at the outflow. From this we can conclude that the chaotic behaviour experimentally found by Chrutchfield et al /2/ should result from turbulences at the outflow.

* part of Ph. D. thesis

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