STABLE STATIONARY DENDRITIC PATTERNS WITH MINIMAL DISSIPATION

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Abstract: If the geometry of the interface between a good electric conductor and a poor conducting material shows a special dendritic structure, the dissipation is minimal.

1. Introduction

Dendritic structures represent ramified, simple connected patterns. In the case of open systems, dendritic structures are often formed that are self-similar over a certain range. The basic structures of roots, tree branches or the neural and vascular system, for example, can be described as self-similar dendritic patterns. In the anorganic nature dendrites appear in electric discharges, in the formation of river systems or in aggregation und crystallization processes (Feuerecker 1987, Georgii 1987, Pietronero 1986, Sander 1987).

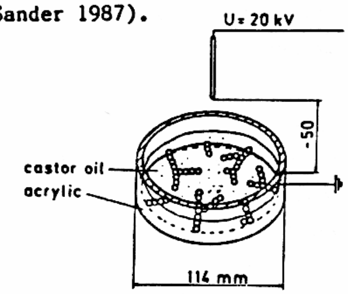

Fig. 1
Experimental set-up: charges are sprayed on the oil-surface and transported to the grounded border by conducting particles forming dendrites.

2. Experiment

In order to study properties of dendritic structures, we have performed the following experiment (fig.1): The set-up used consists essentially of a cylindrical cell, made up by a 3 mm layer of castor oil (because of its high dielectric constant) within an acrylic dish of 114 mm diameter and a 47 mm layer of air above. The inner perimeter of the dish is equipped with a grounded metal ring. The potential between a metallic tip and this ring is 20 kV. Charges are sprayed quasi homogeneously upon the oil-surface. Inside the dish there are N metallic bearing balls of 2 mm diameter, which undergo a certain arrangement in order to transport the charges to the metal ring. The experiment was done several times for four different numbers of balls (N = 400, 600, 800, 1000), the dish has been photographed for each (fig.2a and b). It was observed that for a voltage superior to 25 kV the endpoints of the dendrites are lifted to the oil-surface (fig.2c). The experiment shows also that the structure formed is stable: displacing some balls slightly from their positions, they will instantaniously return to their original places.

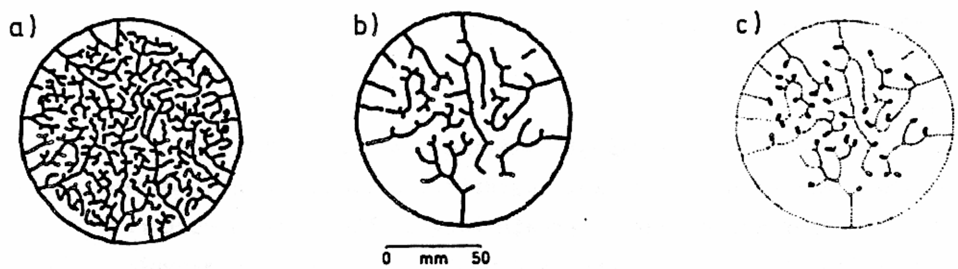

Fig. 2 The dendritic structure developped for
a) 1000 and b) 400 bearing balls
c) shows the same structure as b), but with all points going up to the surface marked.

3. Theory

The two-dimensional description yields that, because of charge conservation, the number of charges sprayed on the oil-surface is equal to the divergence of the current $\underline{j}$:

$$\text{div}\ \underline{j} = -\ Q(\underline{x}) - dn(\underline{x})/dt \quad ,$$

where $\underline{j}(\underline{x})$ is the electric current, $Q(\underline{x})$ the source-strength and $n(\underline{x})$ the charge-concentration of the electric charges at the position $\underline{x}$ on the oil-surface. As the current performed is small, the magnetic field can be neglected and the electric potential U may be deduced from OHM's law

$$s_o . \underline{j} = -\ \text{grad}\ U,$$

where s_o is the specific resistance of the oil; the resistance within the particles can be neglected. The electrical relaxation time being significantly inferior to the mechanical relaxation time, the positions of the particles approach time independent values. This final stationary state is characterized by minimal potential energy V, since V is a LIAPUNOV function. The dissipation D due to ohmic losses is proportional to V, according to

$$D = \int_{\text{oil}} d\underline{x}\ (\underline{E}^2/s_o) = V/(s_o e),$$

and is also a minimum; e is the dielectric number.
In our experiment we considered only the final, stationary arrangement of the balls. Then the total resistance must be minimal. We studied the total ohmic resistance as a function of the particle number.

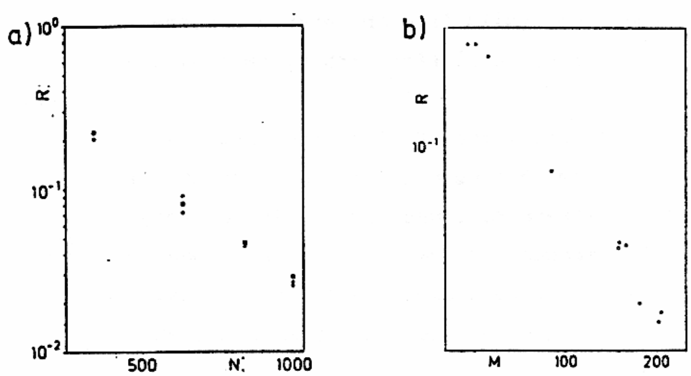

Fig. 3 The total resistance R as a function of
a) particle number N: $R \sim N^C$, with $C = 0.73 \pm 0.03$.
b) the number M of endpoints.

4. Discussion

In order to evaluate the total resistance of the system, the positions of the balls, directly or indirectly in contact to the ground, have been digitized, fed to computer and set to potential equal to zero. For each point (x,y) the potential U(x,y) has been evaluated by solving the POISSON equation by a numerical relaxation method. From fig.3a it can be

seen that the system's resistance R decreases with increasing particle number N by a power law $R \sim N^{C}$, with $C = 0.73 \pm 0.03$. From the calculated potential U(x,y) we have evaluated the divergence of the electric field at each point, with the result that the divergence is extremal at the endpoints of the dendrites. A ball with only one neighbour is called an endpoint, all other balls are called connecting points. Since the divergence at the endpoints is much higher than for all other points, we conclude that the charges sprayed upon the oil-surface are essentially collected by the endpoints of the dendrites. An experimental demonstration of that fact is described above (section 2; fig.2c).
Dendritic structure may be interpreted in the following way: the distribution of charges being homogeneous, the endings of the dendrites, collecting those charges from the surface, will also be distributed homogeneously; we assume that the other balls form the shortest connection to carry off charges to the grounded metal ring with minimal dissipation.
From fig.3b it can be seen that the resistance R also decreases with increasing number M of endpoints. From this we can conclude that stable structures, exhibiting minimal resistance R, cannot form closed loops, because an infinitesimal gap would produce two new endpoints attached to the ground by connecting balls, and therefore diminish the total resistance. This would be in contradiction to the fact that the resistance has been a minimum already before.

Literature

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*: part of Ph.D. thesis