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Resonant Stimulation of Nonlinear Damped Oscillators by Poincaré Maps

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Abstract: A new method for resonant stimulation of nonlinear damped oscillators by nonlinear entrainment is presented. Appropriate driving forces are calculated with Poincaré maps. Without using any feedback, these resonant driving forces are in phase with the velocity of the oscillator and cause a huge energy transfer.

1. Introduction

Physical systems with a marked nonlinearity and chaotic solutions can be found in various fields of physics/1/. In general the corresponding differential equations are not integrable/2/. If the dynamics is composed of a smooth oscillation and a comparatively slow modulation of the amplitude of this oscillation, the essential properties of the dynamics can be described by maps, e.g. Poincaré maps and stroboscopic maps/3/. These maps can be much simpler than the differential equation, can easily be solved numerically, and are mathematically well examined/4/. Recently methods have been presented to calculate these maps numerically/5/ and analytically/6/. The aim of this paper is to show, that those maps can be used to stimulate nonlinear oscillators resonantly.
It has been shown analytically /7/ and experimentally/8/ that nonlinear oscillators can be stimulated resonantly by nonlinear entrainment. New experiments with the experimental set up of /8/ show, that a resonant stimulation of the nonlinear oscillator is even possible if a rough approximation of the ideal driving force/7/ is used /9/. This rough approximation is constructed by a smooth interpolation between the extrema of the ideal driving force. The extrema of the driving force can be calculated with a special Poincaré map. The time between the extrema is the recurrence time of the Poincaré map.

2. Resonant stimulation of nonlinear oscillators

As a physical system we consider a damped nonlinear oscillator of the following type:

$$\ddot{y} + \eta_1 \dot{y} + K_1(y) = F(t) \tag{1}$$

where y is the amplitude, η_1 a friction constant, K_1 is a nonlinear force and F(t) represents an external perturbation. We assume that the experimentalist has the following simplified/10/ model of the unperturbed system:

$$\ddot{z} + \eta_1 \dot{z} + K_2(z) = 0 \tag{2}$$

In order to calculate a resonant driving force F(t), the following differential equation has to be numerically integrated:

$$\ddot{x} + \eta_2 \dot{x} + K_2(x) = 0 \tag{3}$$

According to /7/ the perturbation

$$F(t) = -2\, \eta_2\, \dot{x} \tag{4}$$

is resonant for $\eta_2 = -\eta_1$, i.e. (3) results from (2) by a reflection of time, and for $K_2 = K_1$, i.e. the model is exact. Since y(t)=x(t) is a special solution of (1),

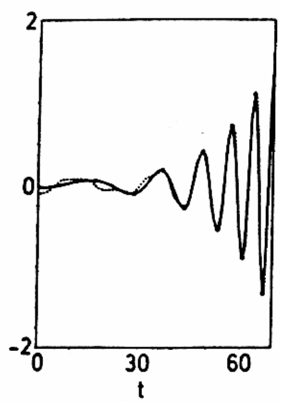

Table 1 Coefficients $c_{1,i}$ and $c_{2,i}$

i	$c_{1,i}$	$c_{2,i}$
0	-0.01	1.2
1	0.53	-0.52
2	0.71	-0.018
3	-0.71	-0.040
4	0.35	-0.016
5	-0.06	-0.002

Fig.1 Resonant stimulation of the oscillator $\ddot{y} + 0.1\,\dot{y} + y^3 = 0$ with an approximation F'(t) of the resonant driving force F(t). Plotted is $\dot{y}(t)$ (dotted line) and F'(t)/0.2 (straight line). The energy transfer is large, because $\dot{y}(t)$ and F'(t) are in phase. F'(t) is calculated by a numerical spline interpolating between the extremas of F(t). The extremas are calculated by an backward iteration of the Poincaré map $P_{\dot{y}}$. The Poincaré map $P_{\dot{y}}$ and the recurrence time T are numerically approximated with

$P_{\dot{y}} : \dot{y}_{n+1} = (-1)^n \sum_{i=1}^{5} c_{1,i} |\dot{y}_n|^i$, $T_n = \exp(\sum_{i=1}^{5} c_{2,i} (\log|\dot{y}_n|)^i)$. The coefficients $c_{1,i}$ and $c_{2,i}$ are listed in table 1.

(3) is called aim differential equation. When the dynamics of the unperturbed experimental system y(t) and the dynamics of the aim equation x(t) are represented in a phase space/3/ the corresponding trajectories have the same geometry. If the dynamics of the unperturbed experimental system is composed of a smooth oscillation and a comparatively slow modulation of the amplitude of this oscillation, the resonant driving force F(t) has the same property, because of (4). In this case, the dynamics of the unperturbed experimental system, aim differental equation, and the resonant driving force can be approximated by an interpolation scheme based on the extrema $\dot{y}_n$, $\dot{x}_n$, and F_n of the velocity $\dot{y}(t)$, $\dot{x}(t)$, and the driving force F(t). The extrema $\dot{y}_n$, $\dot{x}_n$, and F_n are calculated by the Poincaré maps $P_{\dot{y}}$, $P_{\dot{x}}$ and P_F. The time between the extrema is the recurrence time $T(\dot{y}_n)$. The Poincaré map of the extrema of the resonant driving force P_F is proportional (4) to the backward iteration of $P_{\dot{y}}$, if $P_{\dot{y}}$ is invertible. Otherwise P_F can be calculated from $P_{\dot{x}}$ using (4). There are some analytical and numerical methodes to estimate $P_{\dot{x}}$, $P_{\dot{y}}$, and T/5,6/(fig.1).
The resonant driving process is stable, if

$$\ddot{\varepsilon} + \eta_1 \dot{\varepsilon} + \frac{dK_1}{dy}\Big|_{x(t)} \varepsilon = F'(t) - F(t) \qquad (5)$$

is asymptotically stable, where F'(t) is the approximated resonant driving force and $\varepsilon = y(t) - x(t) \approx 0$.

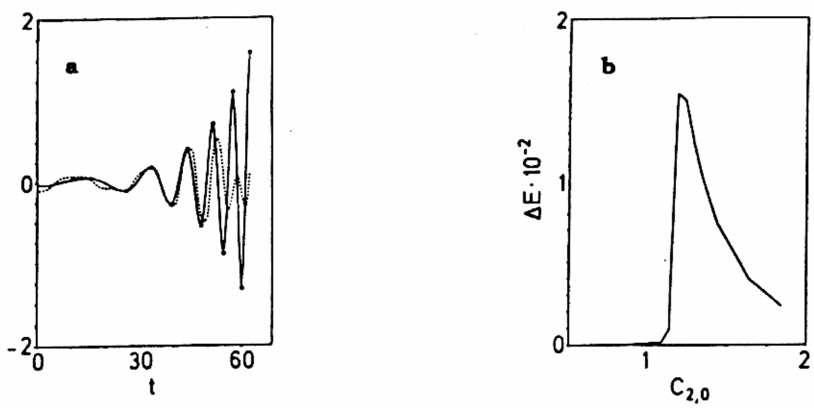

Fig.2 Estimation of a Poincaré map by resonance spectroscopy. (a): If the parameter $c_{2,0}$ of the Poincaré map (Tab.1) is equal to 1.0 no resonant stimulation of the system $\ddot{y} + 0.1\ \dot{y} + y^3 = 0$ is possible. Plotted is $\dot{y}(t)$ (dotted line), $\dot{y}_n$ (o) and F'(t)/0.2 (straight line). $\dot{y}(t)$ and F'(t) are out of phase. (b): The energy ΔE of the same oscillator after ten oscillations of the driving force is largest if $c_{2,0}$ has the value of table 1. The numerical simulations show, that only for this parameter value the resonance condition $\dot{y}(t) \sim F'(t)$ is satisfied.

3. Resonance Spectroscopy with Poincaré maps

If the paramters of the model or the parameters of the Poincaré map P_y are wrong, generally no resonant stimulation is possible/7/. The Poincaré map can be estimated by a systematic variation of the parameters(fig.2).

4. Conclusion

The generalisation of the above techniques to systems of differential equations, and partial differential equations is straightforward/8,11,12/, and its application to controlling/8,11,12/ Navier-Stokes flows or certain equations of biology might have important consequences.

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