

to appear in proc. of int. conf. "spatial inhomogeneities and transient behaviour in chemical kinetics", Brussels, September 1987

Stable Dendritic Structures with Minimal Dissipation

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Abstract: If the geometry of the interface between a good conduction material and a bad conducting material shows a special dendritic structure, the dissipation at the interface is minimal. In several systems this is a stable stationary state.

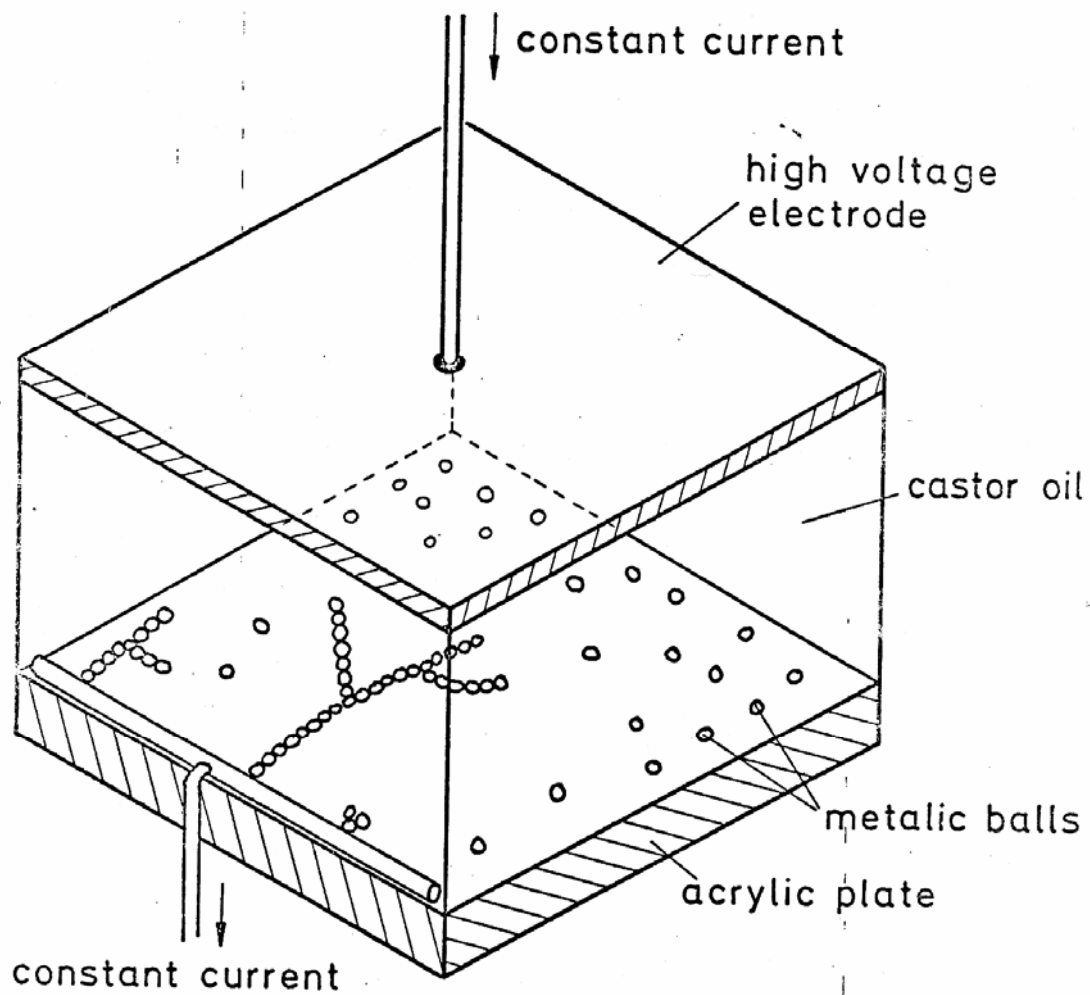
Electric charges are continuously and homogenously spread on a horizontal acrylic plate (20 cm x 20 cm) with a layer of castor oil (5cm)(Fig.1). We investigate the patterns formed by 2000 metallic balls (\emptyset 1mm), which initially are randomly distributed in the castor oil. The balls move according to $\eta \dot{\underline{x}}_i = -dV/d\underline{x}_i$ where \underline{x}_i is the position position of ball #i and η represents a friction constant, until the potential energy V of the system is minimal, if the electronic relaxation time is small compared to the mechanic relaxation time. In this case the charge distribution is slaved by the coordinates of the balls (Haken 1983) and

$V = \int dx \cdot \frac{1}{2} \epsilon E^2(\underline{x})$ is a Liapunov function of the system (Georgii et al. 1987) with ϵ the dielectric constant of castor oil, E : electric field, \underline{x} : position in the oil. In the stable stationary state the dissipation

$$P = \int dx s E^2(\underline{x})$$

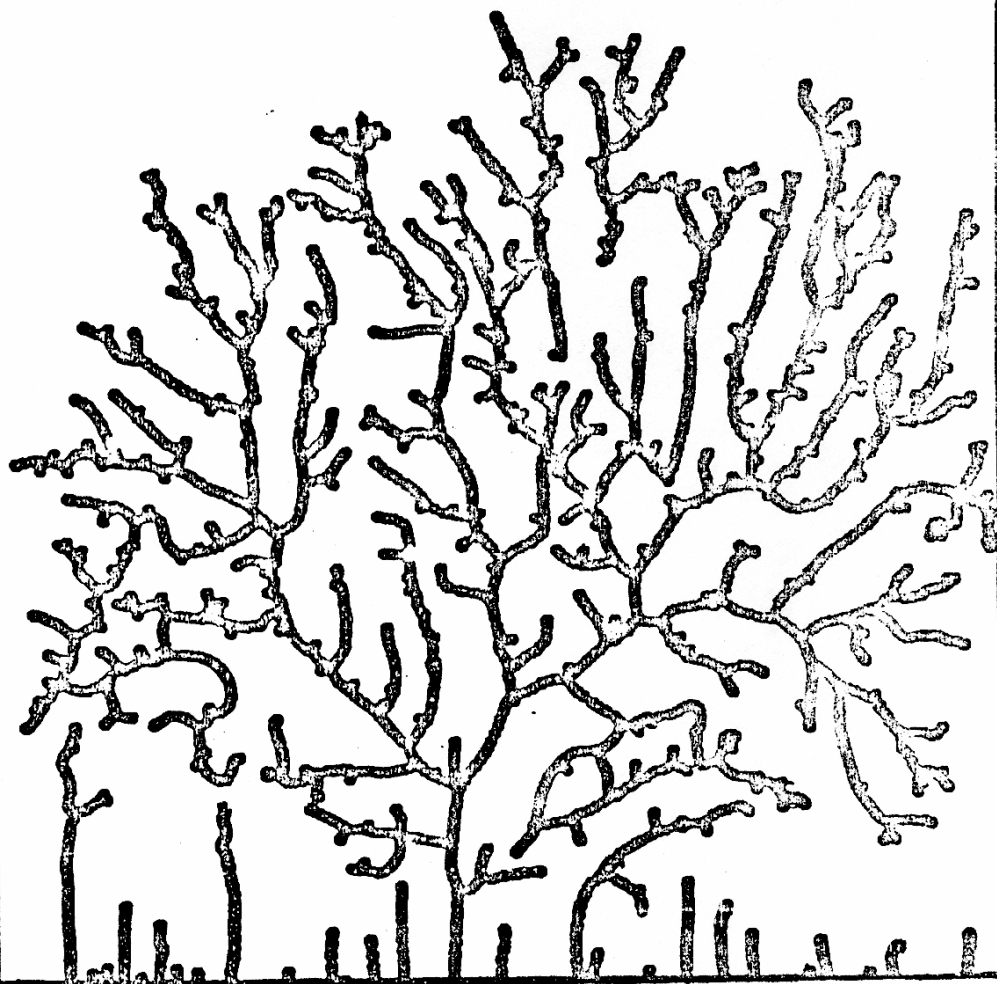
is minimal, where s represents the electric conductivity of castor oil. The resulting stable structures have a minimal dissipation, as the dissipation P is proportional to the potential energy V (Prigogine 1985). The dissipation by Ohmic resistance is an implicit function of the coordinates of the balls and an explicit function of the electric field. Because of the short electric relaxation time the electric field is close to equilibrium and Onsager relations can be used to calculate the dissipation (Prigogine 1985), while mechanic coordinates show cooperative effects. The dissipation by the motion of the balls is neglected.

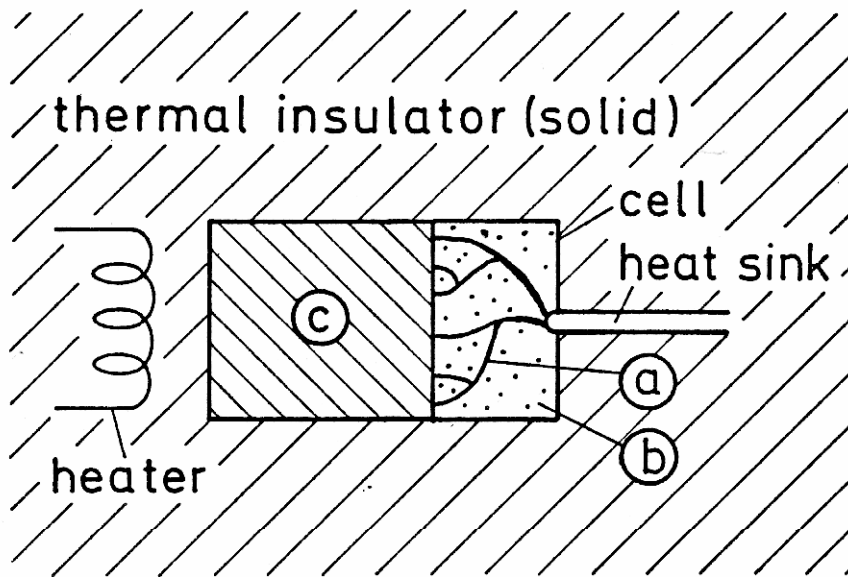
Experiments (Schmöllner et al. 1987, Merté et al. 1988) show (Fig.2) that the resulting stable structures are dendritic (dendritic structures are simple connected structures i.e. structures without closed loops). Also for analog problems with heat conduction (Fig.3), ionic diffusion, and information flow, those structures fulfill an analogue variation principle.



0 mm 100

120 min





- Ⓐ material with high thermal conductivity
- Ⓑ liquid or gas with low thermal conductivity
- Ⓒ solid material with low thermal conductivity

* part of Ph.D.-thesis

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Fig. 1 Experimental set-up

Fig. 2 Metallic balls agglomerate under the influence of electric current and form a dendritic pattern.

Fig. 3 Condensation under the influence of a constant flow of heat. When the good conduction material (a) condenses in a dendritic structure, the temperature in the cell is low, since the average thermal resistivity of the cell is small. Therefore these dendritic structures are stable, when surface tension is small.