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Dendritic Structures in Fluids of High Viscosity*

G. Feuerecker, A. Hübler, and E. Lüscher

Physik-Department, Technische Universität München, D-8046 Garching, Federal Republic of Germany

Abstract. Simple, reproducable dendritic patterns are generated in high viscous fluids and mathematical and computer simulations methods based on hydrodynamical models have been developed.

1 Introduction

In order to describe mathematically the form of natural structures a large number of data have to be analysed. Often the application of techniques from the Differential Geometry is not possible, partially due to lack of periodicity. Also many natural forms cannot be reduced to simpler geometric forms without loss of important information. According to Mandelbrot (1983) "clouds are not spheres, mountains are not cones... nor does lightning in a straight line". The level of complexity which nature exhibits is not simply of a more advanced degree but of an overall different degree. For the description of many of the irregular and fragmental patterns in nature Mandelbrot has introduced the concept of fractal (or Hausdorff) dimension. Some fractal sets are curves or surfaces others are disconnected clusters. Some of the complex patterns can be derivated from the Navier-Stokes equation and the conservation of volume boundary condition and reproduced in straight forward manner with the computer. The fractal dimension can be used as a criterion for the quality of the computer simulation in respect to nature. But we have to keep in mind the words of Leo Kadanoff: "But without organizing principles, the field (fractals) tends to decay into a zoology of interesting specimens and facile classifications. Despite the beauty and elegance of the phenomenological observations upon which the field is based, the physics of fractals is, in many ways, a subject waiting to be born".

2 Experimental Set-Up

A chop of grease (Deltinol NLGI-Grad 2) is placed between two parallel plates which are pressed together in order to create a circular spot with a diameter of the order 3–4 cm. Dendritic structures (Fig. 1) from this circular spot are obtained by pulling the plates with a well defined force between 1 and 200 N.

We are using the fractal dimension D of the borderline for the description of the dendritic structure. The length of the curve is measured with the scaling length m which can be applied N-times:

$$N = L_0 m^{-D}. \tag{1}$$

The fractal dimension is defined by:

$$D = -\frac{d \ln N}{d \ln m}. \tag{2}$$

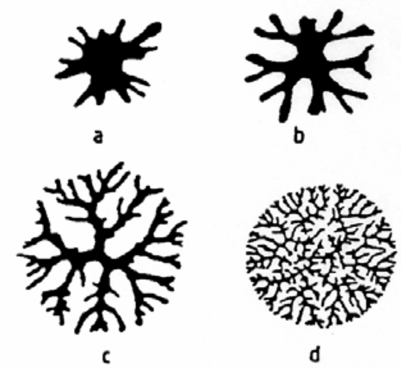

Fig. 1a–d. Experimental dendritic structures obtained with the pull-forces: **a** $909 \frac{\mathrm{N}}{\mathrm{kg}}$, **b** $1270 \frac{\mathrm{N}}{\mathrm{kg}}$, **c** $9230 \frac{\mathrm{N}}{\mathrm{kg}}$, **d** $59\,500 \frac{\mathrm{N}}{\mathrm{kg}}$

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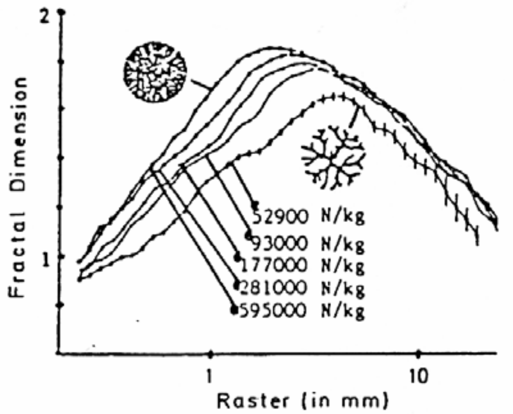

Fig. 2. Fractal dimension $D(m)$ as a function of the scaling length m from the experiment with $h=0.2$ mm and different pull-forces per mass unit as parameter

If the range of linear dimension becomes bigger than the diameter of the fractal structure the dimension tends towards zero with increasing m. For low values of m the dimension reaches $D=0$ because a curve represented by a set of points is again of the dimension zero. In an intermediate range the dimension $D=D(m)$ depends on the form of the pattern-curve. The maximum value of D is characteristic for the patterns. Using a "box-counting-algorithmus" and a fit for the $(\ln N-\ln m)$-values it is possible to obtain reliable values for the dimension $D(m)$ in the range of $\left[3r, \frac{R}{3}\right]$ where r represents the accuracy of the resolving power and R the overall size of the object (Klotz et al. 1983). The fractal dimension D as a function of the scaling length is re-presented in Fig. 2 for different applied forces as a parameter.

With increasing forces the dimension shifts toward smaller scaling length which corresponds to more pronounced dendritic structure. The maximal value D_{max} of the fractal dimension corresponds to the very fine structure of the dendrites. The correlation between D_{max} and the pull-force per mass of the fluid-spot can be represented with the following empirical relation (Feuerecker et al. 1986):

$$D_{max}=2-A\left(\frac{F}{m}\right)^{B}, \tag{3}$$

where

$A=0.65$

$B=-0.3$

$$\left[\frac{F}{m}\right]=\frac{\mathrm{N}}{\mathrm{kg}}.$$

3 Model and Simulation

We consider the 2-dimensional problem (x-y-plane). From the Navier-Stokes equation we get the flow between the two plates neglecting inertial and gravitational forces

$$\mathbf{j}=-\frac{\nabla p}{\mu} \tag{4}$$

with

$$\mu=\frac{12\eta}{h^2}$$

p: pressure

η: kinematic viscosity

h: distance of the plates

$\mathbf{j}$: flow velocity.

The flow in the z-direction is not defined, therefore we represent the z-flow with a source term obtained from the conservation of the fluid volume:

$$\nabla\cdot\mathbf{j}=-\frac{\dot{h}}{h}. \tag{5}$$

The simplified 2-dimensional Navier-Stokes equation becomes [Eq. (5) in (4)]:

$$\nabla\cdot\nabla p=\mu\frac{\dot{h}}{h}=g. \tag{6}$$

The pressure inhomogeneity of this Poisson-equation is independent of x and y and can be evaluated through the dissipation P, F represents the applied force:

$$P=F\cdot\dot{h}=\frac{h}{\mu}\iint(\nabla p)^2\,dx\,dy. \tag{7}$$

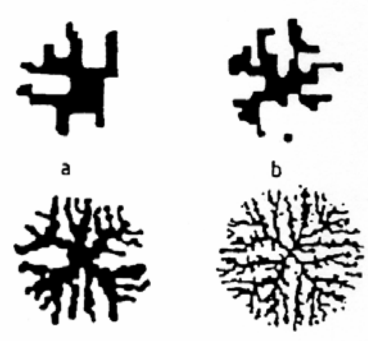

Fig. 3a–d. Computer simulated experiment. The pull-force increases from **a** to **d**; $g=g_0=\frac{8F}{\pi R^4}$

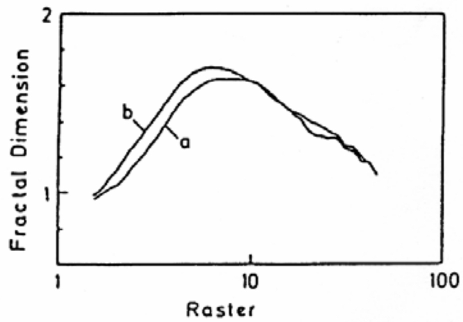

Fig. 4. Fractal dimension $D(m)$ for two pull-forces (a and b) of the simulated experiment

Introducing (7) in (6) one gets

$$g = \frac{1}{F} \iint (\nabla p)^2 \, dx \, dy \tag{8}$$

with the initial value

$$g_0 = \frac{8F}{\pi R4}, \tag{9}$$

where F represents the force and R the radius of the spot.

The boundary condition for the pressure is given by the surface tension. The pressure distribution can easily be computed. The flow velocity at the border follows from (4). We normalized the maximum flow velocity to one pixel per step. The calculated structures are shown in Fig. 3 where the applied pull-force increases from a to d.

The calculated fractal dimension as a function of the scaling length is represented in Fig. 4. Comparison between the Figs. 1 and 2, respectively 3 and 4 demonstrates good agreement between the experimental results and the theoretical model.

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References

Feuerecker G, Hübler A, Lüscher E (1986) Helv Phys Acta 59:128

Klotz K, Hübler A, Lüscher E (1986) Helv Phys Acta 59:124

Mandelbrot BB (1983) The fractal geometry of nature. W.H. Freeman, New York

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Prof. Dr. E. Lüscher
Physik Department
Technische Universität München
D-8046 Garching
Federal Republic of Germany