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Pattern Formation of Powder on a Vibrating Disc*

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Abstract. Fine grain powder placed on a regularly vibrating disk shows dissipative patterns. This behaviour was investigated and a mathematical model was developed. The pattern formation can be explained through a nonlinear diffusion term.

1 Experiment

Jenny observed already in 1967 that powder (Lycopodium) homogeneously placed on a regularly vibrating disk is forming clusters with very sharp boundaries. According to Gierer and Meinhardt (1982) a structuring process out of the homogeneous distribution can be explained with an autocatalytic amplification. An other explanation can be given introducing a nonlinear diffusion term due to collisions between the powder particles which are only partially elastic.

The powder is distributed homogeneously on an acrylic disc mounted on a loudspeaker membran (Fig. 1). The loudspeaker is driven by a sinus generator with frequencies between 50 and 300 Hz. The vibration intensity is measured through the maximal acceleration a of the disc. At the vibration frequency of 85 Hz and an acceleration of $a = 10$ g the typical amplitude is 3×10^{-4} m. A characteristic growth of "little convesting hills" can be observed as shown in Figs. 2 and 3.

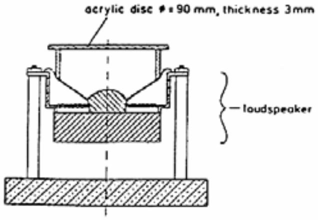

Fig. 1. Experimental set-up. The acrylic disc is coupled to the membran of a loudspeaker

Fig. 2. Lycopodium powder on the disc

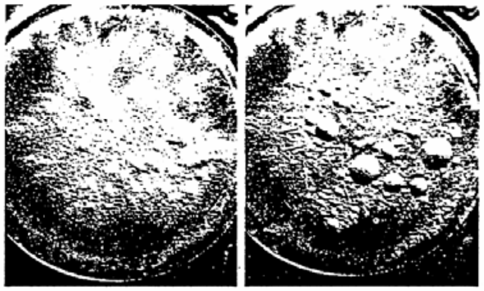

Fig. 3. Pattern formation of the powder (hilly landscape) initially shown in Fig. 2 with $f = 85$ Hz, $a = 9$ g. The typical diameter of a little hill 30 s after the onset of vibration is 10 mm

The evolution of structures are represented in Fig. 4a–e. The "hills" shown in Fig. 4e are in a

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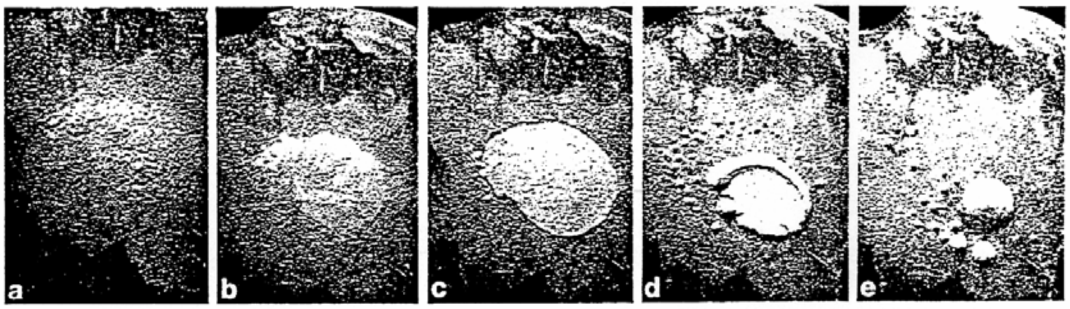

Fig. 4a–e. Sequence of the pattern formation at $f=85$ Hz. **a** $a=3$ g, **b** $a=5$ g, **c** $a=8$ g, **d** $a=10.5$ g, **e** $a=10.5$ g

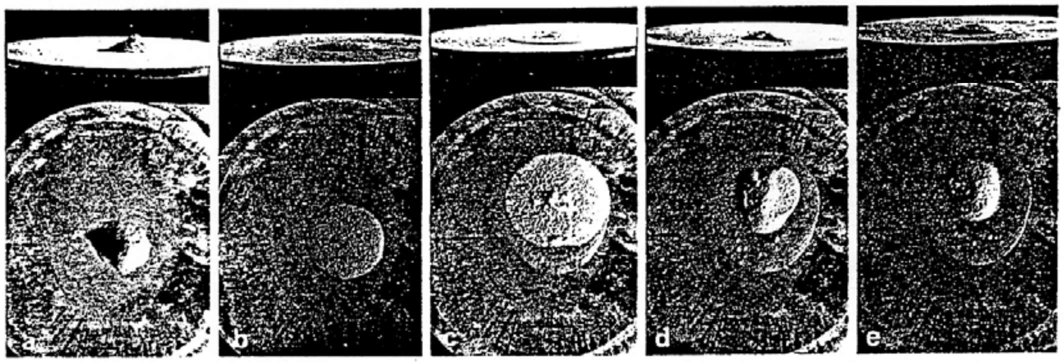

Fig. 5a–e. Sequence of the structuring-expanding and contracting-initially started with a cone shaped distribution at $f=85$ Hz. **a** $a=0$ g, **b** $a=3$ g, **c** $a=8.25$ g, **d** $a=8.25$ g, 10 s later, **e** $a=9.25$ g, 20 s later

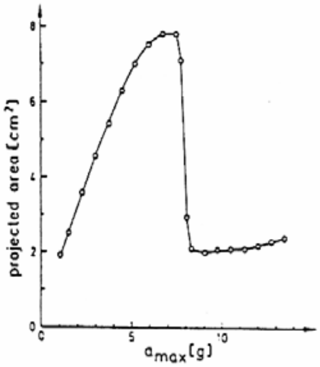

Fig. 6. Projected area of the hilly landscape (starting condition: single cone distribution) as a function of the maximum acceleration. At the acceleration around $a=8$ g a "phase transition" can be observed

continuously convecting motion. The series of pictures Fig. 5a–e demonstrates the development of a cone-shaped initial distribution. The correlation between acceleration a (measure for the amplitude) and the (xy)-extension of the hills for the development of one initial cone-shaped hill (xy-area $2\ \mathrm{cm}^2$) is plotted in Fig. 6. The acceleration was increased by $\Delta a=+0.75$ g every 30 s. There is a very pronounced phase transition at $a=8$ g, the flat big area becomes *a* "little convecting hill" with small (xy)-area extension.

In the series of pictures Fig. 7a–c in the "flat land" in the vicinity of two "hills" the formation of initially two smaller and finally only one additional "hill" can be observed (conditions $f=85$ Hz, $a=9.5$ g, time interval between the pictures 1 s).

The inverse phase transition from "hills to flat land" can be observed at smaller amplitudes and shows a hysteresis represented in Fig. 8. The acceleration was decreased and increased every 15 s for an amount of $\Delta a=\pm 0.75$ g.

In Fig. 9 the result of higher acceleration is shown; the powder convection currents become turbulent and particles from the main hills are ejected and agglomerated to smaller hills which are moving towards the main hill. Similar experiments – only qualitatively up to now – as described above have also been executed in a reduced pressure atmosphere (0.5 mbar) and the observed pattern formation shows a similar phase transition and convection pattern behaviour represented in Figs. 10–12.

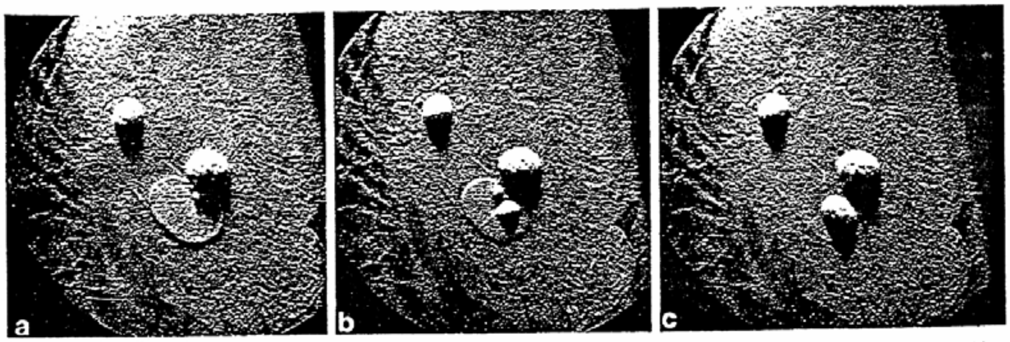

Fig. 7a–c. Sequence demonstrating the transformation from a "flat land" to one hill. $f = 85$ Hz, $a = 9.5$ g. The time interval between the pictures is 1 s. The two initially existing hills to not change their geometry

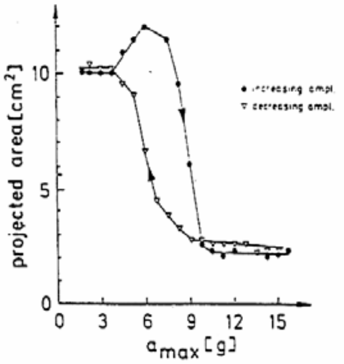

Fig. 8. Observation of a hysteresis effect in the area/acceleration representation, changing the sign of the acceleration

Fig. 10. Behaviour of the powder in a low pressure atmosphere (0.5 mbar), $f = 85$ Hz. The vertical view is a mirror picture. The boundaries are very pronounced

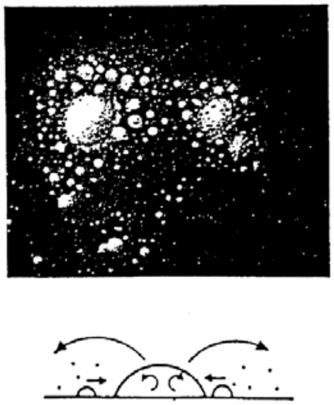

Fig. 9. Behaviour of the particles at higher acceleration. Ejection of particles and formation of secondary hills followed by reunification. On the bottom the simplified model is represented

Fig. 11. Reduced atmospheric pressure condition, $f = 85$ Hz, with 5% higher amplitude than in Fig. 10. Weak structuring is observable

Fig. 12. Reduced atmospheric pressure condition, $f = 85$ Hz. A fine detailed pattern formation is observable. The amplitude is 10% higher than in Fig. 10

2 Essay of a Qualitative Description

We assume for all the hopping powder particles the same mass and indicate their projected positions in the (xy)-plane. The layer thickness d is equal to the number of particles in z-direction at a position x, y. A low particle density ϱ results in a large hopping width and vice versa. The hopping width for the $S(d)$ top particle at a given position can be written as:

$$S(d) = S_0 \exp[C(d-1)],$$

where S_0 and C are constant and $C < 0$.

The vibrating disc accelerates the lowest particle in an ideal case to the maximal disc velocity

$$v_1 = \frac{a}{2\pi f},$$

where a is the maximum acceleration and f the frequency of the disc.

After a partial-elastic collision with an other particle the velocity of the second particle becomes

$$v_2 = \frac{1+k}{2} v_1,$$

where $k = 0$ represents an inelastic and $k = 1$ elastic scattering.

The velocity of a top particle after ($d-1$)-collisions is

$$v_d = v_1 \left(\frac{1+k}{2}\right)^{d-1} = v_1 \exp\left[\ln\left(\frac{1+k}{2}\right) \cdot (d-1)\right].$$

The typical angle of the initial hopping trajectory tangent with z-axis is α, therefore the hopping width neglecting the friction in the air becomes:

$$S = v_d^2 \frac{\sin 2\alpha}{g}$$

and the above introduced constants can be calculated:

$$S_0 = \left(\frac{a}{2\alpha f}\right)^2 \frac{\sin 2\alpha}{g}$$

$$C = 2 \ln \frac{1+k}{2} < 0.$$

The distribution of directions in the xy-plane of the hoppings in the lowest approximation is stochastic. A diffusion term D can be introduced as a product of the hopping rate h and the square of the hopping width ($S(d)$) which depends on ϱ resp. on d. The hopping rate is proportional to the drive frequency f of the disc and is therefore constant:

$$D \sim h \cdot /S(d)/^2$$

$$D \sim h \cdot \Big/ \left[\frac{a}{2\pi f}\right]^2 \frac{\sin 2\alpha}{g} \exp\left[+2 \ln \frac{1+k}{2} (d-1)\right] \Big/^2.$$

Because the exponent is negativ we observe a self-amplification of the inhomogeneous particle distribution. Particles from a high density region (hill region) exhibit a short range hopping width; those particles which are directly in contact with the disc show a large diffusion length which ends finally on a hill.

The tendency of the powder particles in the gravitational field is governed by the energy minimum principle, therefore the final distribution on the disc neglecting static friction and the non-linear diffusion would be a homogeneous coverage. The current density j due to the flattering of the landscape is proportional to grad d.

Two examples of the results from a computer simulation, where the hopping and sliding of particles is calculated, are shown in Fig. 13. The important control parameter for sliding is the difference between the layer height [number of particles at a random chosen point (xy)] compared with the number of particles and his four neighbours. The simulation confirms the formation of an anisotropic distribution starting from an isotropic one. The proposed model and its simulation is able to explain the apparent structure of the powder; it does not give a description of the convection mechanism indide the hills. This convection can be compared with a Bénard-Rayleigh model. Particles sitting directly at the acrylic disc surface absorb more energy from the vibrating disc then particles in higher layers. Higher energy absorption leeds to a stronger movement of the particles at the bottom compared to those closer to the top, which results in an inversion towards an equipartion. A slight amplification of the vibration stimulates a collective convection motion of the powder particles. This can be easily demonstrated with a model-experiment used in

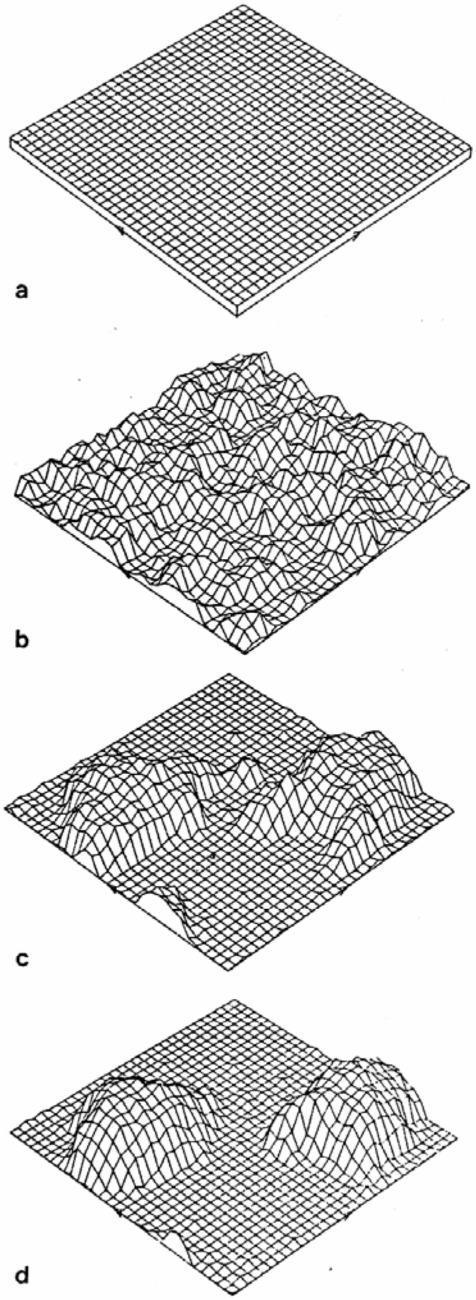

Fig. 13a–d. Computer simulation of the pattern formation (hilly landscape) from an initially "flat country"

Fig. 14. Model-gas molecul demonstration experiment, $f = 69$ Hz. Density inversion from bottom to the top

Fig. 15. Model-gas molecule experiment, $f = 72$ Hz. Convection can be observed

Fig. 16. Pattern formation in fine grain sand on the disc $f = 4$ Hz, amplitude: 6 mm. These apparent patterns are similar to Bénard-structures

introductory physics classes in order to demonstrate the temperature dependence of the movement of gas molecules. The molecules in this model are represented by glass balls. Figures 14 and 15 are pictures taken from this demonstration experiment. According to the absorbed vibration energy the mean free path of the particles at the bottom is higher therefore the density is lower compared to the situation near the top.

Using fine grain sand instead of Lycopodium Bénard-like pattern can be observed, Fig. 16.

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