

# “Homeopathic” Dynamical Systems

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Homeopathy contends that higher dilutions of the active ingredient in a remedy (fewer molecules in each dose) produce stronger medical effects [1]. This idea is inconsistent with observed dose-response relationships of conventional drugs, where the effects increase with the concentration of the active ingredient in the body [2]. The conventional dose-response relationship has been found in numerous scientific studies on organisms as diverse as nematodes [3], rats [4], and humans [5]. Further it is consistent with the behavior of simple physical and chemical systems, -e.g. the larger the force on a spring, the larger the displacement (Hooke’s law), or the larger the voltage on a resistor, the larger the current (Ohm’s law), and the larger the concentration difference within a solution, the larger the particle flux (Fick’s law), the larger the temperature difference on a solid the larger the heat flow, and so on.

The dose-response relationship describes the changes in an organism caused by different levels of exposure (or doses) of a stressor (typically a chemical, heat, radiation, or a force). The dose-response relationship quantifies the response of an individual (e.g. small dose has no observable effect, large dose is fatal) or the response of a population (e.g. how many patients a cured at different levels of exposure). Measuring the dose response relationship, and developing dose response models, are the basis for determining "safe" and "hazardous" levels and dosages for drugs, radiation, potential pollutants, and other substances that humans are exposed to. The dose-response relationship is the scientific basis for public policy.

The dose-response relationship, “the larger the dose, the larger the response”, applies for conventional drugs and simple physical systems.
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A dose-response curve is a graph of the magnitude of the stressor (e.g. concentration of a pollutant, amount of a drug, temperature increase, intensity of radiation, pressure) on a logarithmic scale versus the response of the receptor (e.g. organism, physical system, and ecologic system) in percent. The first point along the graph where the response is non-zero is called the threshold-dose. The steeper the curve raises beyond the threshold-dose, the stronger the active ingredient. Conventional medicine assumes that all active ingredients have a positive strength near the dose-threshold, consistent with the behavior of elementary physical and

chemical systems. However, it is conceivable, that there are more complex systems, where the dose-response curve has a negative slope, and consequently the active ingredient has a negative strength. In the following we will introduce a system where the active ingredient has a negative strength, and illustrate that for such active ingredients, smaller doses produce larger effects, as suggested in Homeopathy.

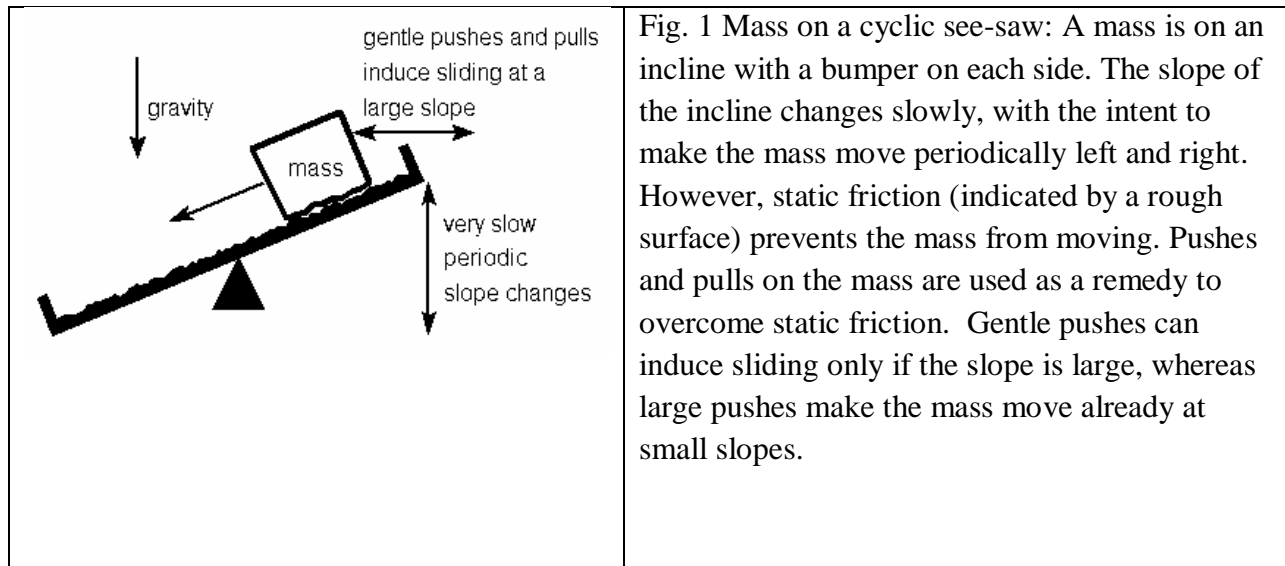


Fig. 1 Mass on a cyclic see-saw: A mass is on an incline with a bumper on each side. The slope of the incline changes slowly, with the intent to make the mass move periodically left and right. However, static friction (indicated by a rough surface) prevents the mass from moving. Pushes and pulls on the mass are used as a remedy to overcome static friction. Gentle pushes can induce sliding only if the slope is large, whereas large pushes make the mass move already at small slopes.

We consider a mass on an incline with bumpers at each end (See Fig. 1). The slope of the incline is changed periodically with the intent to make the mass move left and right in synchrony with the incline.

However, static friction will prevent the mass from sliding, unless the slope exceeds a critical angle, which depends on the friction constant. We study the case where the maximum slope of the incline is slightly less than the critical slope and consequently the mass does not move, as intended.

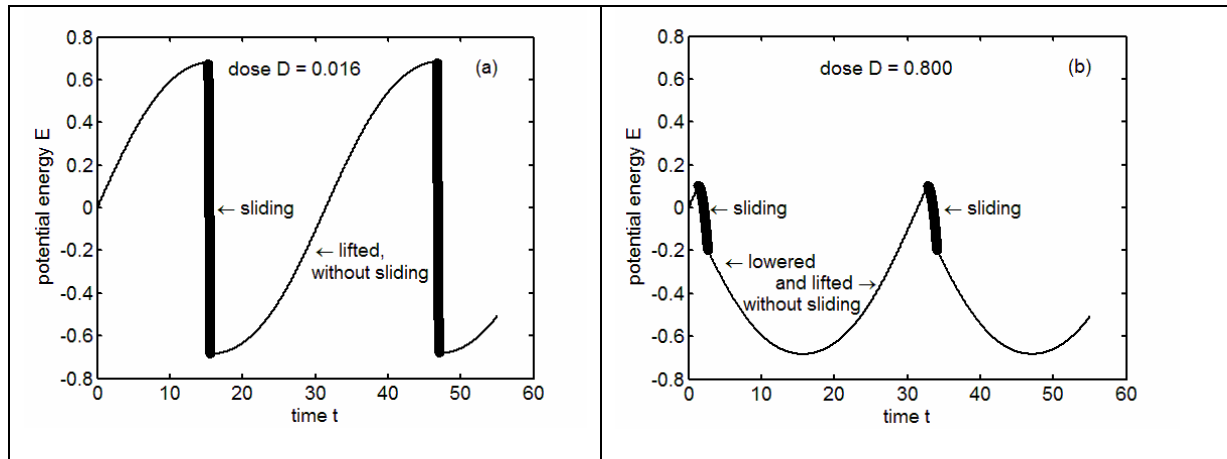


Fig. 2 The potential energy of the mass versus time for gentle pushes (a) and large pushes (b). Thick lines indicate sliding and power consumption. Thin lines indicate lifting and lowering without sliding and without power consumption. Gentle pushes can induce sliding only if the mass is lifted to a large height (a), whereas large pushes make the mass move already at a small height (b). Thus, gentle pushes cause a large power consumption of the cyclic see-saw system, whereas the power consumption due to large pushes is small.

To remedy this problem, we apply some gentle pushes and pulls on the mass, and measure the power dissipated in the resulting slides. Figure 2 shows the potential energy of the mass versus time [6]. When the mass is lifted beyond the horizontal, both, the potential energy increases, and the sensitivity to the pushes increases. If the pushes are large (see Fig. 2b), the mass starts to slide at a small slope, where its position is only slightly above the horizontal, and its potential energy is small. Therefore the energy dissipated in each slide is small. However, if the pushes are small (see Fig. 2a), the mass starts to slide at a much larger slope, from a position high above the horizontal, and its potential energy is large. Therefore the energy dissipated in each slide is large.

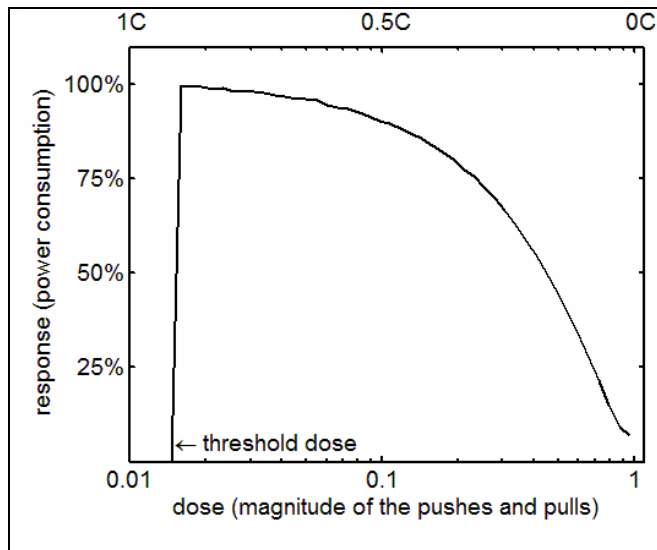


Fig. 3 Dose-response curve of a cyclic see-saw system: The X-axis is the dose on a logarithmic scale (magnitude of the pushes and pulls  $D$ ) and the Y-axis is the response on a percentage scale (power consumption of the see-saw  $P$ ). The slope is negative. Therefore the remedy has a negative strength. Hahnemann's C-scale (top axis) points in the opposite direction. According to the C-scale the slope is positive, and therefore the "homeopathic strength" of the remedy is positive.

The magnitude of the pushes is a measure for the dose of the remedy. The power consumption of the cyclic see-saw system is a measure for the response. Figure 3 shows the dose-response curve of a cyclic see-saw system. The dose-response curve has a dose-threshold. The dose-threshold depends on the system parameters and can be equal to zero [6]. In contrast to the assumptions of conventional medicine, this dose-response curve does not have a positive slope beyond the threshold-dose. At the threshold-dose, the response starts at a high level from where it decreases linearly, *-i.e.*  $P = 100\% - D$ . According to a conventional medicine, the strength of the remedy is the slope of the dose-response curve in a semi-logarithmic plot. In a semi-logarithmic plot, where the scale of the X-axis is  $x = \log(D)$ , the response appears to decrease exponentially, *-i.e.*  $P = 100\% - \exp(2.3 x)$ . Therefore the remedy has a negative strength which diminishes near the threshold-dose. In Homeopathy Hahnemann's C-scale is commonly used to measure doses. Hahnemann's C-scale (see Fig. 3 top axis) is opposite to the conventional measures for doses, *-i.e.*  $c = -0.5 x$ . If the scale of the X-axis is the C-scale, the slope of the dose-response curve positive and, therefore the "homeopathic strength" of the remedy is positive.

Smaller doses produce larger responses in cyclic see-saw systems and other complex physical systems.

Smaller doses produce larger responses in cyclic see-saw systems. Their "homeopathic strength" is positive. This behavior appears to contradict our intuition from simple, passive physical and chemical systems, such a springs, resistors, and ionic solutions. However see-saw systems are more complex, they are cyclic, have a perpetual energy supply, and at certain states of their cycle they are more receptive to perturbations than at other states. Plasma systems and electronic circuits with negative resistors can have a positive "homeopathic strength" as well. This is important, since circuits with negative resistors are the core element of digital computers. Chaotic dynamics might have a positive "homeopathic strength" as well; they are cyclic and their sensitivity to external perturbations depends on their state [7]. In chaotic systems small perturbations tend to grow over time into a large response. It is conceivable that for some chaotic systems the response is even larger if the perturbation is smaller [8]. This would be a "homeopathic", chaotic system, if we call systems with a positive "homeopathic strength" a "homeopathic" dynamical system.

In "homeopathic" dynamical systems the response is larger, if the perturbation is smaller.

Even if most simple physical systems are not “homeopathic”, “homeopathic” dynamical systems may be rather common among complex dissipative systems, including living systems. Recently it has been reported that dividing cancer cells can be destroyed by extremely weak electric fields [9]. If cancer cells are found to be less sensitive to larger electric field, then electric fields have a positive “homeopathic strength” on dividing cancer cells.

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6. The force on the mass is  $F = m g \sin(A) - D \cos(A) \sin(3000t)$ , where  $t$  is time,  $D$  is the dose (magnitude of the pushes and pulls),  $m = 0.1 \text{ kg}$  (mass),  $g = 9.8 \text{ m/s}^2$  (gravity),  $A = A_{\max} \sin(0.1t)$  (angle between the incline and the horizontal),  $A_{\max} = 0.98 A_c$  (maximum angle),  $A_c = \arctan(\mu)$  (critical angle),  $\mu = 1$  (friction constant), and the horizontal and the friction force is  $F_{\mu} = \mu(m g \cos(A) + D \sin(A) \sin(3000t))$ . The mass starts sliding if  $|F| > |F_{\mu}|$  and stops sliding if  $|x| > 1$ , where  $x$  is the position on the incline. The sliding motion is  $x' = -10 m g \sin(A)$ . The potential energy is  $E(t) = m g x \sin(A)$ . The power consumption for each slide is  $P = 100\% \times 2 \max(E(t)) / (2 m g \sin(A_{\max}))$ .
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8. Foster, G.; Hübler, A.; Dahmen, K. Resonant forcing of multidimensional chaotic map dynamics. *Phys. Rev. E* 2007, 75, 036212; in a chaotic systems the typical response is considered to be larger if the initial perturbation is larger, -i.e.  $|\Delta x(t)| = |\Delta x(0)| \exp(\lambda t)$ , where  $|\Delta x(0)|$  is the size of the initial perturbation (the dose),  $|\Delta x(t)|$  is the size of the response, and  $\lambda$  is the largest Lyapunov exponent. The system is chaotic if  $\lambda > 0$ . It is conceivable that the response is smaller if the initial perturbation is larger, such as  $|\Delta x(t)| = (1 - |\Delta x(0)|) \exp(\lambda t)$ .
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