Experiment 1: Chaotic Water Wheel

Background:

- Ed Lorenz's water wheel [E.N. Lorenz, J. Atmos.Sci. 20, 130-141(1993), S.H. Strogatz, Nonlinear Dynamics and Chaos, ISBN 0-7382-0453-6, Perseus Books Publishing LLC (1994)]

- Water Wheel Equation*:*

$$y_{n+1} = (y_n + 2 / \sin y_n) \text{ modulo } (2\pi / B),$$

where y_n is the angle between the vertical and the line through the center of the top bin and the center of the wheel after the *n*-th turn. *B* is the number of bins. This dynamics is deterministic since y_{n+1} is a function *of* y_n. The dynamics is chaotic since the average slope of this function is greater than 1.

If there is static friction, the water wheel has a finite size attractor:

$$y_{n+1} = 0 \quad \text{if } y_n < y_a$$
$$y_{n+1} = (y_n + 2 / \sin y_n) \text{ modulo } (2\pi / B), \quad \text{if } y_n > y_a$$

where y_a is the size of the attractor. The dynamics of this system is very similar to the dynamics of the water wheel equation with "rational" initial condition.

Experiments:

(1) **What kind of motion does the wheel perform as a function of the flow rate?** At which flow rate(s) does the behavior change? Estimate this flow rate with the beaker.

(2) **Adjust the flow so that the turns appear irregular and determine the return map.** Record the angle where the wheel stops twenty times. Plot the y_n-values versus time step *n* and plot y_{n+1} versus y_n. Is the water wheel deterministic? Is the water wheel chaotic?

(3) **Is there a range of y_n-values where the wheel never turns?** Tilt the axis of water wheel by about 30^o with respect to the horizontal. Use the small rock in the container. This increases the static friction and reduces the maximum amount of water in the bins. Now the torque that static friction can produce is comparable in size to the torque due to the weight force of the water in the bins. How large is range of angles y_n where the wheel does not turn? What are the consequences of a finite size attractor?

Experiment 2: Mixing and Segregation in Open Dissipative Systems

Background:

- Experiment: Segregation of materials in a rotating drum (K. Hill, G. Gioia, D. Amaravadi, PRL **93**, 224301 (2004))
- Theory: According to the second law of thermodynamics, disorder, measured in term of entropy, increases in closed systems. Only processes which increase entropy occur spontaneously. This is different in open dissipative systems. In an open dissipative systems order can increase or decrease, depending of the system parameters.

Experiments:

(4) **Circular drum with small and large glass beads (fill level < 50%). What kind of patterns do you observe as a function of the angular frequency?** At which angular frequencies does the behavior change? Estimate the angular frequencies in rotations per second. Sketch the emergent pattern.

(5) **Square drum with small and large glass beads. What kind of patterns do you observe as a function of the angular frequence?** At which angular frequencies does the behavior change? Estimate the angular frequencies in rotations per second. Sketch the emergent pattern.

Experiment 3: Chaotic Heart Beats

Background: Healthy humans have a large variability in the beat-beat intervals of the heart [C.-K. Peng et al, PRL 70, 13431346(1993), M. Costa, A.L. Goldberger, C.-K. Peng, PRL 89, 068102(2002)]. However if the dynamics beat-beat interval changes abruptly, this may indicate an illness too [D. Chialvo, Nature 419, 263 (2002)]. Hence a healthy heart has short term correlations but no long term correlations. Does this mean a healthy heart is chaotic?

Experiment: In this experiment you take an EKG signal of each group member and determine the return map for the beat-beat intervals. Use the return map to determine if the heart beat is chaotic.

A typical heart beat has the extremes labeled P, Q, R, S, and T [see P.E. McSharry, et al., IEEE Transactions on Biomedical Engineering 50, 289-294 (2003)]. The beat-beat interval is the time, t_n between two consecutive R-peaks.

Place the green electrode at right inner arm at the elbow. Place the red electrode at left inner arm at the elbow. Place the black electrode at right inner wrist.

Open the folder CSSS on the Desktop and delete the file *data.txt*. Then start the HBAcquire.vi data acquisition program. Click the Run-button (=>). Record the EKG for 90 seconds. Exit HBAcquire.vi. Make sure that the subject holds still and breathes regularly, otherwise the baseline of the EKG is shifting, which makes the data analysis difficult.

Go to the MATLAB window. Type in “clear” and press ENTER. Then type “EKG” and press enter. This load the “EKG” MATLAB program and executes it. The “EKG” program

- reads the data file,
- plots the time series,
- computes maxima above a certain threshold,
- computes a sequence of time intervals between the extremes t_n
- plot the return map between the extremes

Specific Tasks:

(6) **Record an EKG signal for a group members?** Print the HBAcquire.vi window. How does the recorded EKG signal compare to the theoretical? Adjust the threshold in EKG so that it picks up only R-peaks. Adjust the scale of the window where the R-peaks are labeled and print it. Print the window with the return map of the beat intervals. Is the heart beat at a fixed point?

(7) **Change the MATLAB program so that it picks up the S-minima.** Adjust the scale of the window where the R-peaks are labeled and print it. Print the window with the return map of the beat intervals. Is the heart beat at a fixed point? Is it chaotic? **Extra Credit:** Change the program so that it computes the pulse (average number of beats per minute) and its variance.

Experiment 4: Video Feedback

<u>Background:</u> Video feedback is an example for a two dimensional mapping function with a threshold [J. Crutchfield, PHYSICA 191-205 (1984)]. Systems with auto focus show adaptation to the edge of chaos. [P. Melby, J. Kaidel, N. Weber, A. Hubler, PRL 84 5991-5993(2002)]

<u>Experiment:</u> In this experiment you explore a range of typical video feedback as a function of zoom, rotation, and angle of the camera.

(1) **Put an obstacle in front of the camera.** Change the zoom on the projector. How does the pattern change?

(2) **Change the zoom and the sharpness on the projector?** How does the pattern change?

Experiment 5: Growth and Decay of Fractal Networks

Background: Ilya Prigogine received the Nobel Prize in 1976 for his observation that stable stationary states of certain open dissipative systems are minimum-entropy-production states. An example is the aggregation of conducting particles in Castor oil under the influence of an electric current [J. Jun, A. Hubler, PNAS 102, 536-540 (2005), M. Sperl, A. Chang, N. Weber, A. Hubler, Phys. Rev. E 59, 31653168(1999)]

Experiment: This experiment shows the growth of ramified transportation networks with minimum entropy production (resistance) for three different types of initial conditions.

(1) **Produce a random distribution of beads, then increase the voltage and observe the growth process?** (a) Record the limiting structure with the copy machine (cover the dish). Repeat the experiment and record the limiting structure with the copy machine. How reproducible is the number of tips and branching points? (b) Are there any closed loops? Make a closed loop and check if it opens. (c) Are the resulting structures stable? Break one of the chains (voltage off) and watch it repair (voltage on). (d) Do the trees repel each other? (e) Does the network disintegrate when the voltage is off? How does it decay?

(2) **Place all the beads in the center and repeat the experiment 1 part (a).**

(3) **Place all the beads near the perimeter and repeat the experiment 2.**

(4) **Lower the high voltage electrode into the oil (red location) and remove the circular electrode. Then position the grounded electrode at two different places in the oil (black locations) and let the system grow "wires".** Record the limiting structure with the copy machine.

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(5) **Place a clean Petri dish on the overhead projector and pour a droplet of Motor Cycle oil in the center of the dish. Lower the high voltage electrode into the droplet and charge the droplet.** Monitor the growth of the network.

Experiment 6: Time Discrete Models for Time Continuous Systems

Background: Stephen Wolfram promotes the idea that discrete model provide good models for dynamical systems.

Newton second law for the center of mass provides a time continuous model: $m\,x'' = m\,g_e$ (free fall), and $v = -v$ *if* $x=0$ (collision), where m is the mass of the two-cart system and x is the center of mass. x'' is the acceleration of the center of mass. $g_e = g\,sin(A) \approx g/100$ is the effective gravitational constant. A is the angle between the air track and the horizontal. The continuous model predicts that the collision is infinitely short and that the peak positions of the center of mass decrease monotonically.

When the above equation is discretized with Euler method with a time step T that equals the period of oscillation of the oscillation of the displacement between the two cart, we obtain a time discrete model: x" is replaced by $(x(t+T) - 2\,x(t) + x(t-T))/T^2$ where t is time. In this model the duration of the collision is of length T/2 and the dynamics can be irregular similar to the dynamics of the experimental system.

Experiment: This experiment illustrates the dynamics of a bouncing ball in slow motion. A map is a good model if the spring is not very stiff.

(1) **Place the two carts at the center of the air track and observe a sequence of collisions.** How often does the lower cart collide with the bumper? Record the peak positions of the upper car. Are they monotonically decreasing?

(2) **Study the motion when the amplitude is small.** There are two types of motion: (i) both cars are moving and (ii) only the upper car is moving. The time discrete model predicts the complicated two-car motion and the transition the one-car motion, whereas the time continuous model predicts a simple motion.

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Experiment 8: Oscillating Chemical Reactions

Background: The BZ reaction is one of the most well known chemical oscillators.
[J.M. Bodet, C. Vidal, A. Pacault, F. Argoul, in Non-Equilibrium Dynamics in Chemical Systems, ed. C. Vidal and A. Pacault, Springer Series in Synergetics, 102-107(1984)]

Objective: This experiment illustrates spatially inhomogeneous chemical reactions. Get an intuition on the dynamics of chemical waves.

Procedure: **Use safety goggles and gloves. Start with a small empty Petri dish. Add 3 units of 0.45 molar H_2SO_4. Add 2 units of 0.55 molar Malonic Acid. Add 2 units of 1 molar $NaBrO_3$. Add 9 drops of Ferroin. Mix the liquids by gently moving the dish horizontally on a circular path.**

Data collection:

(1) What happens during the first 60 seconds. Describe your observations.

(2) When do the first wave fronts occur?

(3) How fast do the wave fronts move?

(4) What occurs at the interface fronts?

Experiment 9: Thermo-Acoustic Resonator

Background: A thermo-acoustic resonator converts thermal energy into sound. A flame is placed near the end of a vertical circular tube. A sound wave in the tube makes the flame flicker. The flickering flame strengthens the sound wave. The sound wave is particularly strong if the flame is placed just barely inside the tube. However if the sound wave is strong the draft of the air may blow out the flame.

Objective: This experiment illustrates the paradigm “the hole is more than the sum of the parts”. The sound wave and its effect on the flame depend mostly on macroscopic quantities, such as the shape of the container and the location of the openings, and much less on the microscopic properties of the tube material or the gas inside the tube.

Procedure: Ignite the grill lighter, hold it vertically, and insert the flame into the tube. Avoid heating the glass!

Data collection:

(1) Move the flame slowly in and out. Describe your observations.

(2) Move the flame fast in and out. Describe your observations.

Experiment 10: Elementary Cellular Automata

Background: Elementary Cellular Automata are dynamical systems which are discrete in time, discrete in amplitude, discrete in space, and local. $x_i(t)=0,1$ is the amplitude at time step t at location i, where $t=0,1,2,\ldots,T$ and $i=1,2,\ldots,N$. T is the number of time steps and N is the number of cells. Since CAs are local they obey the following rule:

$$x_i(t+1)=f(x_{i-1}(t), x_i(t), x_{i+1}(t))$$

For more details see: S. Wolfram, A New Kind of Science.

Objective: This experiment illustrates system which is well described by a CA. Further there is a hardware implementation of a CA.

Procedure 1: Poor a small amount of Castor oil on the lower plate. Apply 10kV on the capacitor. Use the construction glove to reduce the distance between the plates
(E. Lüscher, A. Hubler, Helv.Phys.Acta **57**, 264-267(1984)).

Data collection:

(5) Monitor the oil columns as the plates separate slowly. Describe your observations.

Procedure 2: Use the hardware implementation of the CA. The CA has $N=6$ cells, $T=10$ time steps and circular boundary conditions: $x_1(t+1)=f(x_N(t), x_1(t), x_2(t))$**and** $x_N(t+1)=f(x_{N-1}(t), x_N(t), x_1(t))$.

Data collection:

(1) What are the patterns for the following initial conditions:

111111   100000   100001   111000

(2) Which initial condition produces the pattern

??????
111111
000000
000000
000000
. . .

(3) What is the function f?

Experiment 11: Resonances

<u>Background:</u> A dynamical system reacts most sensitive to forcing functions which complement is natural dynamics (separation between neighboring trajectories). This can be achieved by (i) coupling it to a virtual system (see Experiment 4), (ii) by coupling it to complimentary system, - *i.e.* a hard nonlinear oscillator with soft nonlinear oscillator, or (iii) by coupling it to it physical mirror image (this experiment).

For details see: G. Foster, A. Hubler, *Robust and Efficient Interaction with Complex Systems*, Proceedings of 2003 IEEE International Conference on Systems, Man & Cybernetics, 2029-2034(2003).

<u>Objective:</u> This experiment illustrates the interaction of a charged sphere with its mirror image. The interaction with the mirror image is sharp, the interaction is particularly strong.

<u>Procedure:</u> Place the conducting sphere in front of the mirror. Charge the sphere and monitor its motion.

<u>Data collection:</u>

(1) Increase the voltage from 15kV to the maximum value. Describe your observations. Is the sphere attracted to its mirror image? Is the limiting state an stationary, periodic, or chaotic?

(2) Apply 25kV and change the distance between the sphere and the mirror. Describe your observations. Is the sphere attracted to its mirror image? Is the limiting state an stationary, periodic, or chaotic?

Experiment 12: Swarms

Background: Swarms have been studied numerically, but there are very few experimental studies.

Objective: This experiment illustrates herding a swarm.

Procedure: **Use a marker to gather the particles and then use the marker to move them together along a circular path.**

Data collection:

(1) Arrange the particles in a linear chain. Move them slowly/medium speed/fast. Describe your observations.

(2) Arrange the particles in a square pattern. Move them slowly/medium speed/fast. Describe your observations.

(3) What other patterns are stable?

Experiment 13: Solitons

<u>Background:</u> In spatially extended nonlinear systems, such as a chain of pendulums, small amplitude waves move at a constant speed and become smaller as a function of time. Large amplitude waves are complicated. Some of them can be solitons. A soliton is a wave where the amplitude stays constant. In a chain of pendulums, there are solitons and anti-solitons. They are called Sine-Gordon solitons. Sine-Gordon solitons are used as a model for matter and anti-matter.

<u>Objective:</u> Study the creation and dynamics of Sine-Gordon solitons.

<u>Procedure:</u> Use the soliton machine to watch the creation, motion, and anhilation of solitons.

<u>Data collection:</u>

(1) Create a soliton and an anti-soliton in the center of the chain, by rotating the center of the chain horizontally by 360 degrees. Describe your observations. Is the an attractive force between the soliton and the anti-soliton.

(2) Create another soliton and an anti-soliton pair in the center of the chain, by rotating the center of the chain horizontally by 360 degrees again. Describe your observations. Is there an attractive force between two solitons.

(3) Move a soliton or an anti-soliton towards the open end of the chain. What happens to the soliton/ anti-soliton?Is an attractive force between a soliton and the end of the chain?

(4) Move a soliton close to an anti-soliton. What happens to the soliton and the anti-soliton?